# Fractionally Spaced Equalization and Frequency Diversity Methods for Block Transmission with Cyclic Prefix

Yuki Yoshida, Kazunori HAYASHI, and Hideaki SAKAI Dept. of Systems Science, Graduate School of Informatics Kyoto University Yoshida Honmachi Sakyo-Ku, Kyoto, Japan yyoshida@sys.i.kyoto-u.ac.jp, {kazunori, hsakai}@i.kyoto-u.ac.jp

*Abstract*—Frequency domain equalization (FDE) using cyclic prefix has been drawing much attention due to its simplicity and robustness to frequency selective fading channels. Recently, a fractionally spaced equalization method for the FDE system has been proposed by Vaidyanathan and Vrcelj. In this paper, we extend the method to general oversampling factor cases and derive optimal equalizer weights based on Zero-forcing (ZF) and minimum mean-square-error (MMSE) criteria. Furthermore, we show that the proposed method can also be used for an efficient frequency diversity scheme.

# Keywords- cyclic prefix, frequency domain equalization, fractionally spaced equalization, frequency diversity

# I. INTRODUCTION

Frequency domain equalization (FDE) using cyclic prefix (CP) has been drawing much attention due to its simplicity and robustness to frequency selective fading channels [1]. So far, most of the existing FDE systems, such as orthogonal frequency division multiplexing (OFDM) [2], discrete multitone (DMT) [3], or single carrier block transmission with cyclic prefix (SC-CP) [4], utilize a symbol spaced equalizer (SSE) in order to compensate distorted received signals due to the frequency selective fading channels. Recently, a fractionally spaced equalizer (FSE) has been applied to the block transmission systems using cyclic prefix by Vaidyanathan and Vrcelj [5]. However, in the method, the oversampling factor is fixed to be 2 (namely, T/2-FSE, where T denotes the symbol duration) and only ZF criterion is used for the derivation of the equalizer weights.

In this paper, we extend the method to the general case of the T/K-FSE, where K is a general positive integer, and derive optimum frequency domain one-tap equalizer weights based on ZF and MMSE criteria. Here, it should be noted that the FSE with the configuration of the one-tap FDE can not be an optimum MMSE equalizer, in general, while the optimum MMSE SSE can be achieved with the one-tap FDE if the channel noise is white. This is because, in the FSE receiver, the channel noise is no more white due to the receiving (Rx) filter. However, the simplicity of the one-tap FDE is still atractive and we show that the proposed MMSE one-tap FDE can achieve almost the same bit error rate (BER) performance as the optimum MMSE equalizer. Furthermore, we also show that the proposed method can also be used for an efficient frequency diversity scheme. Finally, we demonstrate the effectiveness of the proposed methods via computer simulations.

Bold capital letters will be used to denote column vectors or matrices. An  $M \times M$  identity matrix will be denoted by  $\mathbf{I}_M$  and all-zero matrix of size  $M \times N$  will be denoted by  $\mathbf{0}_{M \times N}$ . We will use  $E[\cdot]$  to denote ensemble average, tr{ $\cdot$ } for trace,  $\|\cdot\|$  for Euclidean norm,  $(\cdot)^{H}$  for Hermitian transpose,  $(\cdot)^{T}$  for transpose, and  $(\cdot)^{*}$  for complex conjugate.

# II. PROPOSED FSE FOR THE SC-CP SCHEME

Figs.1 and 2 show the configuration and the block diagram of the SC-CP scheme with the proposed T/K-FSE with the onetap FDE. In Fig. 2, s(n) denotes an  $M \times 1$  vector of the *n*-th input signal block,  $\mathbf{r}(n)$  is a KM×1 vector of received signals after oversampling, and  $\hat{s}(n)$  is an  $M \times 1$  vector of the equalizer output. The T/Kspaced oversampling and corresponding downsampling at the receiver can be expressed as the expander operation (inserting K-1 zeros between two neighboring symbols) at the transmitter and the decimator operation (picking up every K-th symbols) at the receiver in the discrete time signal model [6]. In the block transmission settings, the expander is given by a  $KM \times M$  matrix U, whose (K(m-1)+1)-th row  $(m=1, \dots, M)$  is equal to the *m*-th row of  $I_M$  and the other rows are the zero row vectors of  $\mathbf{0}_{1 \times M}$ . For example,

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \tag{1}$$

where K=2 and M=3. On the other hand,  $\mathbf{U}^{\text{H}}$  stands for the decimator. **H** denotes a *KM*×*KM* circulant channel matrix and the first column is given by  $\mathbf{h}=[h_{c}(0), h_{c}(T/K), h_{c}(2T/K), \cdots,$ 



Figure 2. The block diagram of SC-CP scheme with the proposed one-tap T/K-FSE

 $h_c$  (*KL*·*T*/*K*), 0, ..., 0 ]<sup>T</sup>, where  $h_c(t)$  denotes the impulse response of the channel including the transmitting (Tx) and Rx filters and *L* is the order of the response.  $W_M$  denotes the *M*-point DFT matrix where the (*m*, *n*) element (*m*, *n* = 1, 2, ..., *M*) is given by

$$\mathbf{W}_{M}(m,n) = \frac{1}{\sqrt{M}} e^{-2\pi j(m-1)(n-1)/M},$$
(2)

and  $\mathbf{W}_{M}^{H}$  is the *M*-point IDFT matrix.

The circulant matrix **H** can be diagonalized by pre- and post- multiplication with *KM*-point DFT and IDFT matrices [7], i.e.,  $\mathbf{H} = \mathbf{W}_{KM}^{H} \mathbf{\Lambda}_{h} \mathbf{W}_{KM}$ , where  $\mathbf{\Lambda}_{h} = \text{diag}\{\lambda_{h}(1), \lambda_{h}(2), \cdots, \lambda_{h}(KM)\}$  is a *KM*×*KM* diagonal matrix whose diagonal elements are the *KM*-point DFT of **h**. Thus  $\mathbf{r}(n)$  can be written as

$$\mathbf{r}(n) = \mathbf{W}_{KM}^{H} \mathbf{\Lambda}_{h} \mathbf{W}_{KM} \mathbf{Us}(n) + \mathbf{n}(n), \qquad (3)$$

where  $\mathbf{n}(n)$  is a *KM*×1 noise vector. The equalizer output  $\mathbf{\hat{s}}(n)$  is given by

$$\hat{\mathbf{s}}(n) = \mathbf{U}^H \mathbf{W}_{KM}^H \mathbf{\Lambda}_f \mathbf{W}_{KM} \mathbf{r}(n), \qquad (4)$$

where  $\Lambda_f = \text{diag}\{ \lambda_f(1), \lambda_f(2), \dots, \lambda_f(KM) \}$  is a  $KM \times KM$  diagonal matrix of the FSE weights.

# III. SIMPLIFICATION OF THE PROPOSED T/K-FSE

In this section, we simplify the configuration for the proposed *T/K*-FSE with one-tap FSE. Since post-multiplication with U is the operation to extract every *K*-th columns of the multiplied matrix, the (m, n) element of  $\tilde{\mathbf{U}} := \mathbf{W}_{KM}\mathbf{U}$  is given by

$$\widetilde{\mathbf{U}}(m,n) = \frac{1}{\sqrt{KM}} e^{-j \cdot 2\pi \cdot (m-1)(K(n-1)+1-1)/KM}, \\
= \frac{1}{\sqrt{K}} \cdot \frac{1}{\sqrt{M}} e^{-j \cdot 2\pi \cdot (m-1)(n-1)/M}, \\
= \frac{1}{\sqrt{K}} \cdot \mathbf{W}_M(l+1,n),$$
(5)

where  $l := m-1 \pmod{M}$ ,  $m = 1, 2, \dots, KM$ , and  $n = 1, 2, \dots, M$ . Thus, we have

$$\tilde{\mathbf{U}} = \frac{1}{\sqrt{K}} \begin{bmatrix} \mathbf{W}_M \\ \vdots \\ \mathbf{W}_M \end{bmatrix} = \frac{1}{\sqrt{K}} \begin{bmatrix} \mathbf{I}_M \\ \vdots \\ \mathbf{I}_M \end{bmatrix} \mathbf{W}_M.$$
(6)

From (3) and (6), we can rewrite the equalizer output  $\hat{s}(n)$  as



Figure 3. The block diagram of the proposed T/K-FSE with one-tap FDE

$$\hat{\mathbf{s}}(n) = \frac{1}{\sqrt{K}} \mathbf{W}_{M}^{H} \begin{bmatrix} \mathbf{\Lambda}_{f1} & \mathbf{\Lambda}_{f2} & \dots & \mathbf{\Lambda}_{fK} \end{bmatrix} \mathbf{W}_{KM} \mathbf{r}(n), \quad (7)$$

where  $\Lambda_{fk} = \text{diag} \{ \lambda_{f1}((k-1)M+1), \dots, \lambda_f(kM) \}$  ( $k=1, 2, \dots, K$ ) is an  $M \times M$  submatrix of  $\Lambda_{f}$ . Therefore, the configuration of the T/K-FSE can be redrawn as in Fig. 3. Furthermore, defining an  $M \times M$  submatrix  $\Lambda_{hk} = \text{diag} \{ \lambda_h((k-1)M+1), \dots, \lambda_h(KM) \}$  ( $k=1, 2, \dots, K$ ), we have

$$\hat{\mathbf{s}}(n) = \frac{1}{K} \mathbf{W}_{M}^{H} \Big[ \mathbf{\Lambda}_{f1} \mathbf{\Lambda}_{h1} + \dots + \mathbf{\Lambda}_{fK} \mathbf{\Lambda}_{hK} \Big] \mathbf{W}_{M} \mathbf{s}(n) \\ + \frac{1}{\sqrt{K}} \mathbf{W}_{M}^{H} \Big[ \mathbf{\Lambda}_{f1} \dots \mathbf{\Lambda}_{fK} \Big] \mathbf{W}_{KM} \mathbf{n}(n).$$
(8)

# IV. DERIVATION OF THE OPTIMUM FSE WEIGHTS

In this section, we derive the ZF and MMSE based equalizer weights of the proposed T/K-FSE with the one-tap FDE. Moreover, we also show the optimum MMSE T/K-FSE weights for comparison purpose.

### A. Proposed ZF T/K-FSE with one-tap FDE

First, we derive the ZF criterion-based equalizer weights for the proposed T/K-FSE with the one-tap FDE. The ZF condition is given by  $\hat{\mathbf{s}}(n)=\mathbf{s}(n)$  in the absence of noise. Thus, the condition for the proposed FSE is given by

$$\Lambda_{f1}\Lambda_{h1} + \dots + \Lambda_{fK}\Lambda_{hK} = K\mathbf{I}_M$$

$$\iff \boldsymbol{\nu}_m^H \boldsymbol{\xi}_m = K, \quad (m = 1, 2, \dots, M), \qquad (9)$$

where  $K \times 1$  vector  $\mathbf{v}_m = \left[ \lambda_f^*(m) \lambda_f^*(M+m) \cdots \lambda_f^*((K-1)M+m) \right]^T$ 

and  $\xi_m = [\lambda_h(m) \lambda_h(M+m) \cdots \lambda_h((K-1)M+m)]^T$ . When  $K \ge 2$ , (9) has a certain freedom in the choice of  $\mathbf{v}_m$  as shown in [5]. Hence, we also exploit this freedom to minimize the effect of additive noise at the equalizer output, i. e., minimize  $E[||\mathbf{e}(n)||^2]$ , where

$$\mathbf{e}(n) := \frac{1}{\sqrt{K}} \mathbf{W}_{M}^{H} [\mathbf{\Lambda}_{f1} \dots \mathbf{\Lambda}_{fK}] \mathbf{W}_{KM} \mathbf{n}(n).$$
(10)

Denoting the autocorrelation matrix of  $\mathbf{n}(n)$  as  $\mathbf{R}$ ,  $\mathbf{E}[||\mathbf{e}(n)||^2]$  can be calculated as

$$\mathsf{E}[\|\mathbf{e}(n)\|^{2}] = \frac{1}{K} \operatorname{tr} \left\{ [\Lambda_{f1} \dots \Lambda_{fK}] \check{\mathbf{R}} \begin{bmatrix} \Lambda_{f1}^{H} \\ \vdots \\ \Lambda_{fK}^{H} \end{bmatrix} \right\},$$
$$= \frac{1}{K} \sum_{m=1}^{M} \nu_{m}^{H} \check{\mathbf{R}}_{m} \nu_{m}, \qquad (11)$$

where  $\mathbf{\check{R}} := \mathbf{W}_{KM} \mathbf{R} \mathbf{W}_{KM}^{H}$  and the (i, j) element  $(i, j = 1, \dots, K)$  of  $\mathbf{\check{R}}_{m}$  is equal to the (m+(i-1)M, m+(j-1)M) element of  $\mathbf{\check{R}}$ . By minimizing (11) under the constraint (9), the ZF *T/K*-FSE weights is derived as

$$\nu_m = \frac{K \tilde{\mathbf{R}}_m^{-1} \boldsymbol{\xi}_m}{\boldsymbol{\xi}_m^H \tilde{\mathbf{R}}_m^{-1} \boldsymbol{\xi}_m}.$$
 (12)

Eq. (12) shows the capability of the proposed FSE to avoid the noise-enhancement, which deteriorates the BER performance of the SSE based on ZF criterion. For example, if we assume  $\mathbf{R} = \sigma_n^2 \mathbf{I}_{KM}$  (i. e.,  $\mathbf{\check{R}}_m = \sigma_n^2 \mathbf{I}_K$ ) to simplify our discussion, the ZF equalizer weights of the SSE (*K*=1) and the ZF *T*/*K*-FSE can be represented as

(SSE) 
$$\lambda_f(m) = \frac{\lambda_h^*(m)}{|\lambda_h(m)|^2},$$
 (13)  
 $(\frac{T}{K} - FSE) \quad \lambda_f(m) = \frac{K\lambda_h^*(m)}{\sum_{k=1}^K |\lambda_h(l+1+(k-1)M)|^2}.$   
 $(l = m - 1 \pmod{M})$  (14)

When  $|\lambda_h(m)|$  is nearly equal to zero,  $|\lambda_f(m)|$  tends to be extremely large value and amplify the channel noise in the ZF SSE. We do not have this problem for the ZF *T/K*-FSE, since it is unlikely that all  $|\lambda_h(l+1+(k-1)M)|$  (*k*=1, ..., *K*) are nearly equal to zero.

# B. Proposed MMSE T/K-FSE with one-tap FDE

Next, we derive the MMSE criterion-based equalizer weights for the proposed T/K-FSE with the one-tap FDE. The mean-squared error (MSE) is given by

$$MSE := E[||\mathbf{s}(n) - \hat{\mathbf{s}}(n)||^{2}]$$
  
=  $E[tr\{(\mathbf{s}(n) - \hat{\mathbf{s}}(n))(\mathbf{s}(n) - \hat{\mathbf{s}}(n))^{H}\}].(15)$   
Assuming  $E[\mathbf{s}(n)\mathbf{n}(n)^{H}] = \mathbf{0}_{M \times KM}$ , and  $E[\mathbf{s}(n)\mathbf{s}(n)^{H}] = \sigma_{s}^{2}\mathbf{I}_{M}$   
we have

$$MSE = \sigma_{s}^{2} tr\{\mathbf{I}_{M}\} - \frac{\sigma_{s}^{2}}{K} tr\{\mathbf{\Lambda}_{f1}^{H}\mathbf{\Lambda}_{h1}^{H} + \ldots + \mathbf{\Lambda}_{fK}^{H}\mathbf{\Lambda}_{hK}^{H}\} - \frac{\sigma_{s}^{2}}{K} tr\{\mathbf{\Lambda}_{f1}\mathbf{\Lambda}_{h1} + \ldots + \mathbf{\Lambda}_{fK}\mathbf{\Lambda}_{hK}\} + \frac{\sigma_{s}^{2}}{K^{2}} tr\{[\mathbf{\Lambda}_{f1}\mathbf{\Lambda}_{h1} + \ldots + \mathbf{\Lambda}_{fK}\mathbf{\Lambda}_{hK}][\mathbf{\Lambda}_{f1}^{H}\mathbf{\Lambda}_{h1}^{H} + \ldots + \mathbf{\Lambda}_{fK}^{H}\mathbf{\Lambda}_{hK}^{H}]\} + \frac{1}{K} tr\{[\mathbf{\Lambda}_{f1} \ \ldots \ \mathbf{\Lambda}_{fK}]\mathbf{\tilde{R}}\begin{bmatrix}\mathbf{\Lambda}_{f1}^{H} \\ \vdots \\ \mathbf{\Lambda}_{fK}^{H}\end{bmatrix}\}.$$
(16)

Using  $\mathbf{v}_m$ ,  $\boldsymbol{\xi}_m$  and  $\check{\mathbf{R}}_m$ , (16) can be represented as

MSE = 
$$M\sigma_s^2 - \frac{\sigma_s^2}{K} \sum_{m=1}^M \boldsymbol{\xi}_m^H \boldsymbol{\nu}_m - \frac{\sigma_s^2}{K} \sum_{m=1}^M \boldsymbol{\xi}_m^T \boldsymbol{\nu}_m^*$$
  
+  $\frac{\sigma_s^2}{K^2} \sum_{m=1}^M \boldsymbol{\nu}_m^H \boldsymbol{\xi}_m \boldsymbol{\xi}_m^H \boldsymbol{\nu}_m + \frac{1}{K} \sum_{m=1}^M \boldsymbol{\nu}_m^H \tilde{\mathbf{R}}_m \boldsymbol{\nu}_m.$  (17)

Since the optimum  $\mathbf{v}_m$ , which minimizes (17), satisfies  $\partial$ (MSE)/ $\partial \mathbf{v}_m^* = \mathbf{0}_{K \times 1}$ , we have

$$-\frac{\sigma_s^2}{K}\boldsymbol{\xi}_m + \frac{\sigma_s^2}{K^2}\boldsymbol{\xi}_m\boldsymbol{\xi}_m^H\boldsymbol{\nu}_m + \frac{1}{K}\check{\mathbf{R}}_m\boldsymbol{\nu}_m = \mathbf{0}_{K\times 1}.$$
 (18)

Assuming  $\mathbf{\check{R}}_{m}$  is nonsingular and using matrix inversion lemma [7], the MMSE *T/K*-FSE weights is given by

$$\nu_m = \frac{K \tilde{\mathbf{R}}_m^{-1} \boldsymbol{\xi}_m}{\frac{K}{\sigma_s^2} + \boldsymbol{\xi}_m^H \tilde{\mathbf{R}}_m^{-1} \boldsymbol{\xi}_m}.$$
 (19)

### C. Optimum MMSE T/K-FSE

In our approaches, we have assumed to employ the one-tap FDE, i. e.,  $\Lambda_f$  is assumed to be diagonal. This is because one of the major advantages of the block transmission using cyclic prefix is the computational efficiency of the FDE using fast Fourier transformation (FFT) and this edge is largely missed when  $\Lambda_f$  is not diagonal. However, the optimum MMSE FSE is still worth considering, because it may give us the upper bound of the performance improvement of the proposed *T/K*-FSE with the one-tap FDE.

When **R** is nonsingular, by directly working on (3) and (4) without assuming any configuration of  $\Lambda_{\rm f}$ , we have the optimum MMSE *T/K*-FSE weight matrix  $\Lambda_{\rm f}$  as

$$\mathbf{\Lambda}_{f} = K \mathbf{\Lambda}_{h}^{*} \left( \begin{bmatrix} \mathbf{\Lambda}_{h1} \\ \vdots \\ \mathbf{\Lambda}_{hK} \end{bmatrix} \left[ \mathbf{\Lambda}_{h1}^{*} \ \dots \ \mathbf{\Lambda}_{hK}^{*} \right] + \frac{K}{\sigma_{s}^{2}} \mathbf{\tilde{R}} \right)^{-1}.$$
 (20)

In (20), if  $\mathbf{\check{R}}$  is diagonal,  $\Lambda_{\rm f}$  is also diagonal and the optimum MMSE equalizer can be realized with the one-tap FDE. Note that the elements of (20) are identical to that of (19) in such a case. In SSE settings, one may assume the channel noise being white and this general assumption satisfies the condition above. In FSE settings, on the other hand, the noise resulting from passing the white noise process through the Rx filter is generally colored, and the optimum MMSE equalizer is not realized by the one-tap FDE. In Sec. VII, we will show the comparison of the performances by computer simulations.



Figure 4. The configuration of proposed frequency diversity method using T/K-FSE

#### V. EQUALIZER WEIGHTS FOR OFDM SCHEME

So far, we have discussed the T/K-FSE for the SC-CP scheme, however, OFDM is also an important block transmission scheme with the CP.

The block diagram representation of the OFDM scheme with the proposed FSE is given by substituting  $\mathbf{W}_{M}^{H} \mathbf{s}(n)$  for  $\mathbf{s}(n)$  and  $\mathbf{W}_{M} \mathbf{\hat{s}}(n)$  for  $\mathbf{\hat{s}}(n)$  in Fig. 1. Moreover, both of  $\mathbf{W}_{M}^{H}$  and  $\mathbf{W}_{M}$  are unitary matrices. Therefore, we can utilize the same equalizer weights as the SC-CP scheme for the OFDM scheme.

#### VI. FREQUENCY DIVERSITY METHOD USING PROPOSED FSE

Here, we point out another interpretation of the proposed method. Basically, the FSE is used to improve the performance by taking advantage of the received signal components beyond the Nyquist frequency, which is commonly observed in bandlimited signals, such as a raised cosine spectrum pulse [8]. For the proposed FSE system, this is the case when the Tx and the Rx filters have the same bandwidth as the Nyquist rate.

On the other hand, if we set the bandwidth of the Tx and the Rx filters in the proposed system to be twice or more greater bandwidth and use the expander operation to the original symbols, the transmitted signal has multiple copies of the basic spectrum. Moreover, since the proposed T/K-FSE is designed to utilize the received signal components up to Ktimes greater bandwidth than the Nyquist rate, the proposed FSE can achieve not only the equalization but also the diversity combining simultaneously, without any change in the configuration or the equalizer weights. Figure 4 shows the system configuration of the proposed frequency diversity method using the proposed T/K-FSE. Therefore, by just changing the bandwidth of the Tx and the Rx filters, the proposed method can easily realize the frequency diversity.

# VII. PERFORMANCE EVALUATION

In our simulation, we consider 10-path Rayleigh fading channels with an exponentially decaying power profile. We set the channel order L=10, the decaying factor 0.5, and the temporal position of each path is randomly determined and is uniformly distributed within the interval of [0:LT]. We have employed QPSK scheme with coherent detection for modulation/demodulation scheme, and set the block size M=256, the CP length 32 and assumed the channel noise to be AWGN. Also, we have employed a square-root raised-cosine filter for the Tx and the Rx filters with the roll-off factor of  $\alpha =$ 

0.5. The whole response of the channel including the Tx and Rx filters is assumed to be known to the receiver. Following results on BER performance are obtained via 3000 Monte-Carlo realizations.

Figs. 5 and 6 show BER performances of the SC-CP and OFDM schemes using the proposed FSE (the T/2, T/4-FSE with the one-tap FDE and the optimum MMSE T/2-FSE) and the SSE versus the energy per bit to the noise power density  $(E_b/N_0)$ . The bandwidth of the pass band of the filters is set to be the same as the symbol rate. From the figure, we can see that the proposed FSE can improve the performance compared with the SSE on each case. On the other hands, there is no improvement between the proposed T/2 and T/4-FSE with the one-tap FDE. This is because the performance improvement by using the FSE depends on the excess bandwidth of the transmitted signal spectrum. In our situations, the transmitted signal bandwidth is limited by Tx and Rx filters up to 3/2 times greater than the Nyquist rate. Hence, K=2 is enough to perfectly observe the received signal, and we can't expect any further improvement by observing beyond twice the bandwidth. Also the figures show that the proposed MMSE T/2-FSE with the one-tap FDE can achieve almost the same performance as the optimum MMSE T/2-FSE in both the SC-CP and OFDM schemes. One possible reason for this is that, if M is sufficiently large,  $\check{\mathbf{R}}$  can be approximated by the diagonal matrix, and both weights are equivalent to each other. Therefore, we can improve the performance of block transmission systems by the proposed one-tap FSE with its simple implementation with the small loss in the BER performance.

Figs. 7 and 8 also show the BER performances of the SC-CP and OFDM schemes with the proposed frequency diversity methods using the T/2 and T/4-FSE with the one-tap FDE versus  $E_b/N_0$ . Here all the settings are same as in Figs. 5 and 6, except for the pass bandwidth of the Tx and Rx filters. The bandwidth is set to be the symbol rate times *K*. Both in the SC-CP and OFDM schemes, we can see significant improvement in the BER performance due to the frequency diversity effect. This shows that, the proposed frequency diversity method can efficiently provide diversity gain with slight increase in complexity.

# VIII. SUMMARY

We have proposed the T/K-FSE for block transmission with CP systems and shown the effectiveness of the proposed method over the SSE by computer simulations. What is more, we have shown the proposed one-tap frequency domain T/K-

FSE achieves the sufficient performance compared with the optimum MMSE FSE, despite of its low computational complexity. In addition, we have also proposed the simple frequency diversity methods for block transmission with CP systems using the proposed FSE, and revealed its significant performance improvement by computer simulations.



Figure 5. BER performance of SC-CP scheme with the SSE and the proposed *T/K*-FSE in 10-path Rayleigh fading channels versus  $E_b/N_0$  (*K*=2, 4 Tx and Rx filter: symbol rate)



Figure 6. BER performance of OFDM scheme with the SSE and the proposed *T/K*-FSE in 10-path Rayleigh fading channels versus  $E_b/N_0$  (*K*=2, 4 Tx and Rx filter: symbol rate)



Figure 7. BER performance of SC-CP scheme with the SSE and the the proposed frequency diversity method using the *T/K*-FSE in 10-path Rayleigh fading channels versus  $E_b/N_0$  (*K*=2, 4 Tx and Rx filter: *K* times symbol rate)



Figure 8. BER performance of OFDM scheme with the SSE and the proposed frequency diversity method using the *T/K*-FSE in 10-path Rayleigh fading channels versus  $E_b/N_0$  (*K*=2, 4 Tx and Rx filter: *K* times symbol rate)

- A. Peled and A. Ruiz, "Frequency domain data transmission using reduced computational complexity algorithm," *Proc. ICASSP.* pp.964--967,1980.
- [2] Jr. L. J. Cimini, "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. on Commun.*, vol.COM--33, no.7, pp.529--540, 1985.
- [3] J. A. C. Bingham, "Multicarirrer Moudulation for Data Transmission: An idea whose Time Has Come, " *IEEE Commun. Mag.*, vol. 28, pp.5-14, May 1990.
- [4] D. Falconer, S. L. Ariyavisitakul, A. BenyaminSeeyar, and B. Edison, "Frequency Domain Equalization for Single-Carrier Broadband Wireless Systems," *IEEE Commun. Mag.*, vol.40, pp.58–66, 2002.
- [5] P. P. Vaidyanathan, and B. Vrcelj, "Theory of fractionally spaced cyclic-prefix equalizers," *Proc. ICASSP 2002*, vol.2, pp.1277-1280, 2002.
- [6] P. P. Vaidyanathan, "Multirate digital filters, filter banks, polyphase networks, and applications: a tutorial," *Proc. IEEE*, vol. 78, no. 1, pp. 56-93, 1990.
- [7] G. H. Golub and C. F. van Loan, *Matrix Computations*, 3<sup>rd</sup> ed. Baltimore, MD:Johns Hopkins Univ. Press, 1996.
- [8] John G. Proakis, Digital Communications, McGraw-Hill, 3th ed., pp.617-621, 1995.