On the Block Size Design for Single Carrier Overlap Frequency Domain Equalization

Wladimir Bocquet
France Telecom R&D Tokyo
3-1-13 Shinjuku, 160-0022
Tokyo, Japan
Email: bocquet@francetelecom.co.jp

Kazunori Hayashi and Hideaki Sakai
Graduate School of Informatics, Kyoto University
Yoshida-Honmachi, 606-8501
Kyoto, Japan
Email: {kazunori, hsakai}@i.kyoto-u.ac.jp

***Abstract*— In this paper, we propose, based on the analytical analysis of the received signal for single carrier overlap frequency domain equalizer (SC-OFDE) equalization, an optimum design of the overlap frequency domain equalization. From the estimation of the received signal-to-interference-and-noise-ratio (SINR), we propose an algorithm that automatically adapts one important parameter of the OFDE equalization the used part of the equalized signal, denoted the block size, in the specific case of fixed window length. The procedure consists of selecting the window length and then performed the Fourier transform for the associate length. For each element of the orthogonal basis, we estimate the received SINR and finally select the appropriate block size based on the modulation, the channel variation and the accuracy requested by the transmission. Simulation results show that proposed design for the OFDE transmission can adaptively manage the complexity and the performance of the transmission.**

***Keywords*- Single Carrier, Overlap FDE, SINR Estimation.**

I. INTRODUCTION

Since 90's, single carrier with frequency domain equalization (SC-FDE) has been strongly studied and the concept has been drawing much attention due to the effective and the simplicity of the transceiver [1]. When SC-FDE is used with cyclic prefix, not only does SC-FDE outperform OFDM system in the absence of channel coding but also loosens the requirement on the power amplifiers due to the low peak to average power ratios (PAPR) of the transmitted signals. Recently, both orthogonal frequency division multiplexing (OFDM) and SC-FDE have been seen as complementary solutions to each other since OFDM and SC-FDE can coexist in a dual-mode multiple access system [2] allowing some parts of the signal processing to shift from the mobile to the base station (BS) in the uplink transmission. However, GI is considered as a main restriction to achieve high efficiency system transmission. For example, for IEEE 802.11a/g [3]-[4] based transmission, GI represents 25% of the bandwidth occupation [5]. Transmission with insufficient GI insertion has been studied by several research group [6]-[7] and several authors have proposed specific frame structure to reduce the impact of the GI without creating interference [8]-[9]. Several years ago, a general equalizer for multi-carrier (MC) transmission without any guard interval (GI) has been proposed by [10]-[11] and more recently [12] proposed similar concept.

The rest of the paper is organized as follows. In Section II, we show the system description of the SC-OFDE. In Section III, we introduce the SINR estimation and then we describe, in Section IV, the method to efficiently estimate the block size associated the OFDE equalizer. In Section V, numerical results over QPSK and QAM modulations are presented. Finally, conclusions are drawn in Section VI.

II. SYSTEM DESCRIPTION

A. Signal Description

We assume a coherent-spaces receiver front-end, as well as precise knowledge of the signal phase and symbol timing, such that the channel can be approximated by an equivalent, discrete-time, baseband model, where the transmit filter, the channel, and the received filter, are represented by a discrete-time linear filter with finite-length impulse response [13]

$$h_n = \sum_{k=0}^{P-1} \alpha_k \cdot \delta_{n,\delta_k} \tag{1}$$

of length P. The coefficients α_k are assumed to be time-invariant and known at the receiver and δ_k is the corresponding time delay for the k-th path.

Let denote the i-th transmitted signal block of size N as $\mathbf{s}_i^N = [s_{i,0}, s_{i,1}, \cdots, s_{i,N-2}, s_{i,N-1}]^T$. The i-th received signal block, $\mathbf{r}_i^N = [r_{i,0}, r_{i,1}, \cdots, r_{i,N-2}, r_{i,N-1}]^T$ is given by:

$$\mathbf{r}_i^N = \mathbf{H}_0 \cdot \mathbf{s}_i^N + \mathbf{H}_1 \cdot \mathbf{s}_{i-1}^N + \mathbf{N}_i^N \tag{2}$$

where $\mathbf{N}_i^N = [n_{i,0}, n_{i,1}, \cdots, n_{i,N-2}, n_{i,N-1}]^T$ denote the AWGN with 0-means and the variance of σ_n^2, $\mathbf{H}_0$ and $\mathbf{H}_1$ denote the $N \times N$ channel matrices defined as

$$\mathbf{H}_0 = \begin{bmatrix} h_0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & 0 \\ h_L & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \dots & 0 & h_L & \dots & \dots & h_0 \end{bmatrix}, \tag{3}$$

$$\mathbf{H}_1 = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 & \dots & h_1 \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & h_L \\ 0 & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \dots & 0 & 0 & \dots & \dots & 0 \end{bmatrix}, \quad (4)$$

B. Design orientation of SC Overlap FDE transmission

Let first fix M and N respectively the parameters that define the block and the window sizes. In sections 3 and 4, we will describe with precision, the method that we propose to adapt block size for a fixed window size. Then, let define the signal at the output of the frequency domain equalizer. In the case of SC-OFDE, we propose not to insert any cyclic prefix in the data stream transmission. So, information bits are first encoded, modulated and transmit to the air if we only consider the baseband representation. Main differences are placed at the receiving part and the channel compensation needs to be adjusted.

Considering the transmit sequence without any GI and an overlap FDE equalization, output of the equalizer can be described by:

$$\mathbf{y}_i^{OFDE} = \Gamma^{N \to M} \cdot (\mathbf{W}^N)^{-1} \cdot \mathbf{G}^N \cdot \mathbf{W}^N \cdot \mathbf{r}_i^N \quad (5)$$

where the block size extraction is performed by

$$\Gamma^{N \to M} = [\mathbf{0}_{(N-M)/2}, \mathbf{I}_M, \mathbf{0}_{(N-M)/2}], \quad (6)$$

with:

$$W_{n,k} = \exp\{-j\frac{2\pi \cdot k \cdot n}{N}\}, \quad (7)$$

and the frequency domain equalization becomes equal to

$$\mathbf{G}^N = diag[G_0^N, G_1^N, \cdots, G_{N-2}^N, G_{N-1}^N] \quad (8)$$

In addition, $\mathbf{G}^N = \{G_k^N\}_{0 \le k < N}$ denotes the N-point frequency domain compensation without any interference. Channel correction is simply realized by multiplying on each subcarrier the inverse of the channel coefficient in the Zero Forcing (ZF) linear detector. The channel equalization can be described by:

$$G_k^N = \begin{cases} \frac{H_k^{N*}}{|H_k^N|^2} & \text{for ZF Detection} \\ \\ \frac{H_k^{N*}}{|H_k^N|^2 + \sigma_n^2} & \text{for MMSE Detection} \end{cases}, \quad (9)$$

where $\mathbf{H}^N = [H_0^N, H_1^N, \cdots, H_{N-2}^N, H_{N-1}^N]^T$ is the N-point frequency domain representation of the channel response.

III. Estimation of the Received SINR

A. Signal and Interference Representation

It has been shown for both OFDM and SC-FDE signals [2] that the multipath channel in the frequency domain representation mainly affects some specific part of the signal placed at the header and ender parts of the frequency representation of the frame. In other words, the effect of the interferences due to the lost of cyclic convolution property is mainly visible at the two extreme parts of the signal in the frequency representation. Including this property in the design of the equalizer, we propose a method considering N samples, denoted window length at the input and M elements, denoted block size at the output with $M < N$. Let first denotes the received element of the N-point basis representation as:

$$\mathbf{z}^{(i)} = \mathbf{A}^N \cdot \mathbf{x}^{(i)} + \mathbf{B}^N \cdot \mathbf{x}^{(i-1)} + \mathbf{n}^{(i)}, \quad (10)$$

including the element description:

$$\begin{aligned} A_{p,u} = & \frac{1}{N} \cdot \sum_{k=0}^{N-1} G_k \cdot H_k \cdot W_{k,u-p}^{-1} \\ & - \frac{1}{N} \cdot \sum_{k=0}^{N-1} \sum_{\substack{q=0 \\ if \\ u > N-\delta_q - 1}}^{P-1} G_k . h_q \cdot W_{k,u+\delta_q-N-p}^{-1} \end{aligned} \quad (11)$$

with:

$$B_{p,u} = \frac{1}{N} \cdot \sum_{k=0}^{N-1} \sum_{\substack{q=0 \\ if \\ u > N-\delta_q - 1}}^{P-1} G_k . h_q \cdot W_{k,u+\delta_q-N-p}^{-1} \quad (12)$$

Based on the signal representation described below, we propose to evaluate the received SINR as:

$$\beta_v = \frac{E\Big[||\mathbf{A}^N \cdot \mathbf{x}^{(i)}||^2\Big]_v}{E\Big[||\mathbf{B}^N \cdot \mathbf{x}^{(i-1)}||^2\Big]_v + \sigma_n^2} \quad (13)$$

which represents the value of the received SINR on the v-th subcarrier for the N-point frequency domain basis.

Since the data are independent, the SINR expression can be re-written as:

$$\beta_v = \frac{P_s \cdot |\sum_{u=0}^{N-1} A_{v,u}|^2}{P_s \cdot |\sum_{u=0}^{N-1} B_{v,u}|^2 + \sigma_n^2} \quad (14)$$

where P_s is the transmit power per data symbol.

IV. Adaptive design of the SC-OFDE with fixed sliding window length

A. Proposed procedure

Fig. 1 shows the general principle and implementation of the proposed scheme to adaptively select the appropriate block

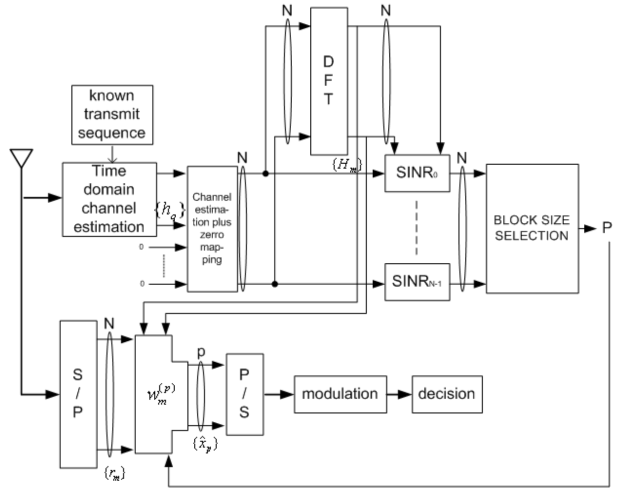

Fig. 1. Proposed scheme

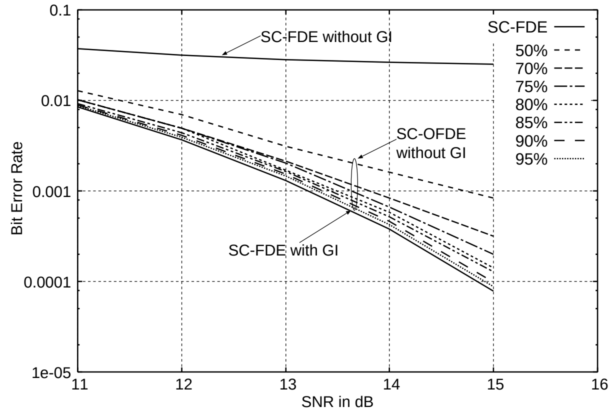

Fig. 2. BER performance for QPSK modulation, R=1 and N=64

size to perform OFDE equalization in a multipath environment. The system consists of first estimating the original transmit SNR from the pilot sequence. Then, we perform the Fourier transform for the fixed window length followed by an estimation of the SINR for each component of the orthogonal basis. The third step consists of evaluating the block size which is associated to the window length the channel variations in function of the transmission request, the original transmit SNR, based on the estimation of the SINR. Once the window length and the associated block size are fixed, we perform the OFDE equalization by simply sliding the window by the size of the associated block. The Fourier transforms conceptually reject the interferences to the extreme parts of the associated window. Basically, OFDE includes this specificity in the way to equalize the received signal.

B. Practical Implementation

In practical implementation, we need to estimate the parameter ξ and the associate value of the block size, M. The block size directly depends on the parameter ξ and the window length N. The proposed estimation of the block size can be described by the following multi-step methods.

First, an estimation of the SINR among all the components, denoted $SINR_m$ of the window slide is performed. Then, we identify the index that satisfies the simple relation:

$$m_{max} = argmax(SINR_{max}) \tag{15}$$

Next, we evaluate the parameter ξ following the requirement in term of allocated modulation, coding rate and requested BER. Practically, ξ can cover a band which includes a certain percentage of variation in term of SINR. Then, we propose to select the block size M following the ξ criteria and the parameter N. Finally, application of the OFDE equalization for the window size N and the block size M.

V. Numerical Experimentation

We now evaluate the performance of the proposed power allocation method for multi-carrier scheme in a mutli-path fading environment. Main simulation parameters are introduced in Table I. We assume perfect knowledge of the channel variations both at the transmitting and receiving parts. For all simulations, a multi-path model is assumed and the carrier frequency is equal to 2.4GHz. For the conventional SC-FDE with (or without) appropriate GI insertion, the IFFT/FFT size is 64 points and the guard interval is set up at 16 samples.

TABLE I
Simulation Parameters

Carrier Frequency	2.4 GHz
Bandwidth	20 MHz
Modulation scheme	QPSK, 16-QAM, 64-QAM
Channel encoder	No code, convolutional codes
Channel estimation	Perfect CSI
Sample period	0.05μs
Number of data packet	30

To verify the behavior of the equalization and selection method based on the estimation of the received SINR, we propose to evaluate the performance for the channel environment described in Table II and for uncoded and coded systems.

TABLE II
Simulation Parameters: Channel model

Number of path	5
Sample period between consecutive paths	0.05μs
Maximum delay spread	0.2μs

For the simulation, we propose to associate the range of SINR with the block size by $\xi = SINR_{max} \cdot (1 - \Gamma_{\%})$, where $\Gamma_{\%}$ is the range of acceptable variation in term of SINR which is accepted.

Figs. 2, 3 and 4 show the BER versus the SINR for three different modulations without any channel encoding scheme. Window size is set-up to 64 and results are presented for

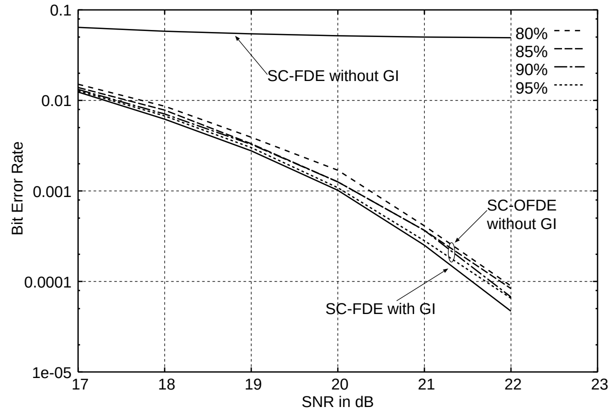

Fig. 3. BER performance for 16-QAM modulation, R=1 and N=64

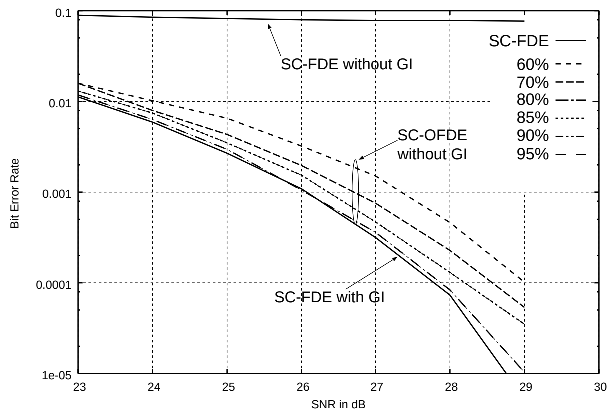

Fig. 4. BER performance for 64-QAM modulation, R=1 and N=64

several values of $\Gamma_{\%}$. In addition, impact of the $\Gamma_{\%}$ value on the block size is also presented in the tables III, IV, V.

TABLE III

IMPACT OF SINR RANGE ON THE BLOCK SIZE FOR QPSK MODULATION

Modulation	Coding Rate	$\Gamma_{\%}$	Block size
QPSK	1	95%	23
QPSK	1	90%	31
QPSK	1	85%	38
QPSK	1	80%	40
QPSK	1	75%	42
QPSK	1	70%	47
QPSK	1	50%	52

Specifically for QPSK modulation, Fig. 2 highlights that impact of $\Gamma_{\%}$ is relatively limited for the range between 90% and 100% in term of BER. BER performance is similar to the specific transmission SC-FDE with appropriate GI. Table III allows us to link the value of $\Gamma_{\%}$ with the block size. For instance, for a value of $\Gamma_{\%}$ equals to 80%, for a window size of 64 elements, we obtain a average block size of 40 elements. In other words, 40 elements of the 64-elements window size get SNIR in the range of the maximum SINR detected within the variation of $\Gamma_{\%}$. When we increase the block size (i.e. the accepted range of SINR becomes more important), significant degradation is obtained. For instance, at BER=10^{-3}, about 1dB degradation is obtained by simply allowing larger range of SINR, $\Gamma_{\%}$=50%

TABLE IV

IMPACT OF SINR RANGE ON THE BLOCK SIZE FOR 16-QAM MODULATION

Modulation	Coding Rate	$\Gamma_{\%}$	Block size
16-QAM	1	95%	21
16-QAM	1	90%	28
16-QAM	1	85%	31
16-QAM	1	80%	34

Figs. 3 and 4 show the BER versus the transmit SNR in dB for different values of $\Gamma_{\%}$. Similar to the QPSK modulation,

TABLE V

IMPACT OF SINR RANGE ON THE BLOCK SIZE FOR 64-QAM MODULATION

Modulation	Coding Rate	$\Gamma_{\%}$	Block size
64-QAM	1	95%	17
64-QAM	1	90%	22
64-QAM	1	85%	25
64-QAM	1	80%	29
64-QAM	1	70%	33
64-QAM	1	60%	38

value of $\Gamma_{\%}$ strongly affects the global BER performances and also the block sizes as described in tables IV and V.

VI. CONCLUSION

This paper proposes an adaptive window size selection for SC-OFDE transmission that adapts the window size (i.e. the complexity of the receiving scheme) in function of the requested BER. The proposed method includes the estimation for the SINR value for each element of the block transmission followed by the estimation of the equalizer coefficients. Basic process consists of evaluating for the entire block, the impact of the IBI and ICI and them estimate the associate power. Then, including the requested BER performance, we propose to automatically select the window size associate to the block size, the modulation and the coding gain. The simulation results have shown validity of the proposed method in term of BER performance for QPSK and QAM modulation and for uncoded and coded systems. Moreover, we would like to emphasize that the proposed method is easily extend to any other advanced equalization for SC-OFDE transmission and any powerful channel encoder such as the convolutional code [16], the turbo codes [17] or the low density parity check codes [18]-[19].

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