A New Beamforming Method for Single Carrier Modulation System with Cyclic Prefix

Kazunori Hayashi, Taku Kojima and Hideaki Sakai Department of Systems Science, Graduate School of Informatics, Kyoto University Yoshida Honmachi Sakyo-ku, Kyoto, 606-8501, JAPAN Email: kazunori@i.kyoto-u.ac.jp

Abstract— This paper proposes a new weight control method for a pre-FFT type adaptive antenna array applied to a single carrier modulation system with cyclic prefix. The weight control method is based on the maximization of the signal-tointerference-plus-noise power ratio (SINR) at the discrete frequency domain equalizer output, therefore, the proposed scheme can increase the received signal power by capturing multiple delayed signals within the guard interval (GI), while avoiding the noise enhancement at the equalizer. Moreover, it can suppress the delayed signals beyond the GI, which is the cause of both the intersymbol interference (ISI) and the interblock interference (IBI). The performance of the proposed method is evaluated via computer simulations, comparing with the performance of weight control methods for the OFDM array and the conventional weight control method.

I. INTRODUCTION

A single carrier modulation system with cyclic prefix (SC-CP) scheme [2]-[4] employs a block transmission with the insertion of a cyclic prefix (CP) as a guard interval (GI), which is also employed in the orthogonal frequency division multiplexing (OFDM)[1] scheme. When delayed signals exist only inside of the GI, the insertion of CP converts the effect of channel from a linear convolution into a circular convolution. Since the circular convolution in time domain corresponds to the multiplication in discrete frequency domain[12], the received signal can be equalized by multiplying complex weights after FFT operation on the signal. The reception of all the delayed signals within the GI can increase the received signal power and it may improve the performance of the SC-CP system, but it also increases the possibility that the channel transfer function has a zero (or zeros) on the FFT grid, which results in serious noise enhancement. We have proposed a multiple delayed signal reception method[5], where weights of a pre-FFT type adaptive antenna array are calculated based on the maximization of the signal-to-noise power ratio (SNR) at the equalizer output. Computer simulation results show that the method can improve the SC-CP system performance significantly.

In the same way as the OFDM system, the performance of the SC-CP system is deteriorated by delayed signals *beyond* the GI, which are the cause of the intersymbol interference (ISI) and the interblock interference (IBI). In this paper, utilizing the interference cancellation ability of adaptive antenna array, we propose a new array weight control method, which not only can receive multiple delayed signals within the GI, while avoiding the noise enhancement at the equalizer, but also can suppress the delayed signals beyond the GI. The proposed method is an extension of the method in [5], and the weight control is based on the maximization of the signalto-interference-plus-noise power ratio (SINR), where the term "interference" includes the ISI and the IBI component after the discrete frequency domain equalization. The performance of the proposed method is evaluated via computer simulations in terms of antenna beam patterns, the SNR at the equalizer output, and the bit error rate (BER), comparing with the performances of weight control methods for the OFDM array and a conventional weight control method.

II. ANALYTIC BER OF SC-CP AND OFDM SYSTEM

It has been shown in [6] that the BER of the binary phase shift keying (BPSK) based SC-CP system BER_{sccp} and the BER of the BPSK based OFDM system BER_{ofdm} can be respectively written as

$$BER_{sccp} = Q\left(\sqrt{\frac{1}{M}\sum_{m=1}^{M}SNR_{m}}\right),\tag{1}$$

$$BER_{ofdm} = \frac{1}{M} \sum_{m=1}^{M} Q\left(\sqrt{SNR_m}\right),\tag{2}$$

where $Q(\cdot)$, M, and SNR_m denote the complimentary error function, the length of the FFT, and the SNR at the *m*-th FFT grid, respectively. (1) and (2) mean that the overall BER of the OFDM system depends on the average BER of the each subchannel, which is determined by the SNR at the each subcarrier, while, in the SC-CP system, the overall BER is determined by the average SNR of each FFT grid. So far, a considerable number of the weight control methods for the OFDM adaptive antenna array [8],[7] have been proposed, however, from (1) and (2), we can see that such methods are not suited for the SC-CP system. The SC-CP system can achieve the best BER performance by maximizing the average SNR at the equalizer output.

III. SYSTEM MODEL

Fig.1 shows the configuration of the proposed system, where a pre-FFT type adaptive antenna array with P elements is followed by a zero-forcing (ZF) based discrete frequency domain equalizer. At time n, the transmitted signal block $\mathbf{s}'(n)$



Fig. 1. Configuration of the SC-CP adaptive antenna array

of size $(M + K) \times 1$ is generated from the information block s(n) of size $M \times 1$ by inserting the CP of K symbols length as the GI.

$$\mathbf{s}'(n) = \mathbf{T}_{cp}\mathbf{s}(n),\tag{3}$$

where \mathbf{T}_{cp} denotes the $(M+K)\times M$ CP insertion matrix defined as

$$\mathbf{T}_{cp} = \begin{bmatrix} \mathbf{I}_{cp} \\ \mathbf{I}_{M \times M} \end{bmatrix},\tag{4}$$

$$\mathbf{I}_{cp} = \begin{bmatrix} \mathbf{0}_{K \times (M-K)} & \mathbf{I}_{K \times K} \end{bmatrix}.$$
(5)

 $\mathbf{0}_{K \times (M-K)}$ is a zero matrix of size $K \times (M-K)$, $\mathbf{I}_{M \times M}$ is an identity matrix of size $M \times M$.

Let L, \mathbf{h}_p , and $\mathbf{n}_p(n)$ denote the channel order, the channel impulse response vector $(M \times 1)$ at the *p*th antenna element $\mathbf{h}_p = [h_{1,p}, h_{2,p}, \dots, h_{L,p}, 0, \dots, 0]^T$, and the noise vector $((M + K) \times 1)$ added to the received signal block at the *p*th antenna element, respectively. Here, $[\cdot]^T$ is the transpose and each element of $\mathbf{n}_p(n)$ is assumed to be the zero-mean white noise with the variance of σ^2 . At the array output, the channel impulse response vector $\mathbf{h} = [h_1, h_2, \dots, h_L, 0, \dots, 0]^T$ and the noise vector $\mathbf{n}'(n)$ are written as

$$\mathbf{h} = \mathbf{H}\mathbf{w}, \quad \mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_P], \tag{6}$$

$$\mathbf{n}'(n) = \mathbf{N}(n)\mathbf{w}, \quad \mathbf{N}(n) = [\mathbf{n}_1(n), \dots, \mathbf{n}_P(n)], \quad (7)$$

where $\mathbf{w} = [w_1, w_2, \dots, w_P]^H$ and $[\cdot]^H$ denote the array weight vector and Hermitian transpose, respectively.

The received signal block $\mathbf{r}'(n)$ is given by

$$\mathbf{r}'(n) = \mathbf{H}_0 \mathbf{s}'(n) + \mathbf{H}_1 \mathbf{s}'(n-1) + \mathbf{n}'(n).$$
 (8)

Here, \mathbf{H}_0 and \mathbf{H}_1 are $(M+K) \times (M+K)$ channel matrices defined as

$$\mathbf{H}_{0} = \begin{bmatrix} h_{1} & 0 & 0 & \dots & 0 \\ \vdots & h_{1} & 0 & \dots & 0 \\ h_{L} & \dots & \ddots & \dots & \vdots \\ \vdots & \ddots & \dots & \ddots & 0 \\ 0 & \dots & h_{L} & \dots & h_{1} \end{bmatrix}, \quad (9)$$
$$\mathbf{H}_{1} = \begin{bmatrix} 0 & \dots & h_{L} & \dots & h_{2} \\ \vdots & \ddots & 0 & \ddots & \vdots \\ 0 & \dots & \ddots & \dots & h_{L} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & 0 \end{bmatrix}. \quad (10)$$

After discarding the CP from the received signal block $\mathbf{r}'(n)$, we have the received signal block $\mathbf{r}(n)$ of size $M \times 1$,

$$\mathbf{r}(n) = \mathbf{R}_{cp}\mathbf{r}'(n)$$

= $\mathbf{R}_{cp}\mathbf{H}_{0}\mathbf{T}_{cp}\mathbf{s}(n)$
+ $\mathbf{R}_{cp}\mathbf{H}_{1}\mathbf{T}_{cp}\mathbf{s}(n-1) + \mathbf{R}_{cp}\mathbf{n}'(n),$ (11)

where \mathbf{R}_{cp} denotes the $M \times (M + K)$ CP discarding matrix defined by

$$\mathbf{R}_{cp} = \begin{bmatrix} \mathbf{0}_{M \times K} & \mathbf{I}_{M \times M} \end{bmatrix}. \tag{12}$$

IV. PROPOSED ARRAY WEIGHT CONTROL METHOD

In this section, we consider the array weight control method when L > K (however L < M), namely, there exist some delayed waves beyond the GI. In this case, since $\mathbf{R}_{cp}\mathbf{H}_0$ in (11) is no longer a circulant matrix[10] and $\mathbf{R}_{cp}\mathbf{H}_1$ is not a zero matrix, it is impossible to eliminate the ISI and IBI by the ZF based channel equalization in the discrete frequency domain.

In order to express the residual ISI and IBI components after the ZF based channel equalization, we separate the matrix H defined by (6) into two $M \times P$ matrices; the component from the channel response within the GI H^{*in*} and the component from the outside the GI H^{*out*},

$$\mathbf{H} = \mathbf{H}^{in} + \mathbf{H}^{out},\tag{13}$$

$$\mathbf{H}^{in} = \begin{bmatrix} h_{1,1} & \dots & h_{1,P} \\ \vdots & \dots & \vdots \\ h_{K,1} & \dots & h_{K,P} \\ 0 & \dots & 0 \\ \vdots & \dots & \vdots \\ 0 & \dots & 0 \end{bmatrix},$$
(14)
$$\mathbf{H}^{out} = \begin{bmatrix} 0 & \dots & 0 \\ \vdots & \dots & \vdots \\ 0 & \dots & 0 \\ h_{K+1,1} & \dots & h_{K+1,P} \\ \vdots & \dots & \vdots \\ h_{L,1} & \dots & h_{L,P} \\ 0 & \dots & 0 \end{bmatrix}.$$
(15)

The received signal block after discarding the CP can be written as

: 0

$$\mathbf{r}(n) = \mathbf{Cs}(n) + \mathbf{C}_{ISI}\mathbf{s}(n) + \mathbf{C}_{IBI}\mathbf{s}(n-1) + \mathbf{n}(n),$$
(16)

where

$$\mathbf{C} = \operatorname{Cir}[\mathbf{H}^{in}\mathbf{w}],\tag{17}$$

: 0

$$\mathbf{C}_{ISI} = \mathbf{R}_{cp} \mathbf{H}_0 \mathbf{T}_{cp} - \mathbf{C},\tag{18}$$

$$\mathbf{C}_{IBI} = \mathbf{R}_{cp} \mathbf{H}_1 \mathbf{T}_{cp},\tag{19}$$

$$\mathbf{n}(n) = \mathbf{R}_{cp} \mathbf{n}'(n). \tag{20}$$

Cir[a] denotes a circulant matrix whose first column is the vector **a**. Here, we also have

$$\mathbf{C}_{ISI} + \mathbf{C}_{IBI} = \operatorname{Cir}[\mathbf{H}^{out}] = \mathbf{D}^{H} \mathbf{\Gamma} \mathbf{D}.$$
 (21)

The ZF based channel equalization is achieved by

$$\mathbf{T} = \mathbf{C}^{-1} = \mathbf{D}^H \mathbf{\Lambda}^{-1} \mathbf{D}, \qquad (22)$$

therefore, the received signal block after the channel equalization becomes

$$\tilde{\mathbf{r}}(n) = \mathbf{s}(n) + \mathbf{T}\mathbf{C}_{ISI}\mathbf{s}(n) + \mathbf{T}\mathbf{C}_{IBI}\mathbf{s}(n-1) + \mathbf{T}\mathbf{n}(n).$$
(23)

Starting from the left, each term in the right side of (23) denotes the desired signal component, the residual ISI component, the residual IBI component, and the noise component, respectively. In this paper, we treat the residual ISI and IBI components as the interference signal, and derive the array weight control algorithm, which maximizes the SINR at the equalizer output, or rather say, minimizes the interference-plus-noise power at the equalizer output.

For the derivation of the cost function, it is necessary to calculate the interference signal power at the equalizer output, however, the interference signal includes two different information signal blocks, namely $\mathbf{s}(n)$ and $\mathbf{s}(n-1)$. Focusing attention to the interference components in $\tilde{\mathbf{r}}(n)$ and $\tilde{\mathbf{r}}(n+1)$ caused by the *n*th information block $\mathbf{s}(n)$, we can see that $\mathbf{s}(n)$ causes the ISI in $\tilde{\mathbf{r}}(n)$ as $\mathbf{TC}_{ISI}\mathbf{s}(n)$ and the IBI in $\tilde{\mathbf{r}}(n+1)$ as $\mathbf{TC}_{IBI}\mathbf{s}(n)$. The sum of the ISI and the IBI components caused by $\mathbf{s}(n)$ in the two consecutive received signal block $\tilde{\mathbf{r}}(n)$ and $\tilde{\mathbf{r}}(n+1)$ can be written as

$$\mathbf{i}(n) = \mathbf{T}(\mathbf{C}_{ISI} + \mathbf{C}_{IBI})\mathbf{s}(n).$$
(24)

In this paper, we consider the power of $\mathbf{i}(n)$ at the equalizer output.

From (21) and (22), i(n) can be written as

$$\mathbf{i}(n) = \mathbf{D}^H \mathbf{\Lambda}^{-1} \mathbf{\Gamma} \mathbf{D} \mathbf{s}(n).$$
(25)

therefore, the interference signal power at the equalizer output becomes

$$P_{i} = E \left[\operatorname{tr}[\mathbf{i}(n)\mathbf{i}(n)^{H}] \right]$$

= tr[$\mathbf{D}\mathbf{\Lambda}^{-1}\mathbf{\Gamma}\mathbf{\Gamma}^{H}\mathbf{\Lambda}^{-H}\mathbf{D}^{H}$]
= $\sum_{m=1}^{M} \frac{\mathbf{w}^{H}\hat{\mathbf{R}}_{m}\mathbf{w}}{\mathbf{w}^{H}\tilde{\mathbf{R}}_{m}\mathbf{w}},$ (26)

where tr[A] and $E[\cdot]$ denote the trace of matrix A and the ensemble average operation, respectively. Also, $\tilde{\mathbf{R}}_m$ and $\hat{\mathbf{R}}_m$ are defined as

$$\begin{split} \tilde{\mathbf{R}}_m &= \tilde{\mathbf{h}}_m \tilde{\mathbf{h}}_m^H, \quad \left[\tilde{\mathbf{h}}_1, \dots, \tilde{\mathbf{h}}_M\right] = (\mathbf{D}\mathbf{H}^{in})^T, \\ \hat{\mathbf{R}}_m &= \hat{\mathbf{h}}_m \hat{\mathbf{h}}_m^H, \quad \left[\hat{\mathbf{h}}_1, \dots, \hat{\mathbf{h}}_M\right] = (\mathbf{D}\mathbf{H}^{out})^T. \end{split}$$

TABLE I

System parameters	
Mod./Demod. Scheme	QPSK/Coherent Detection
Block Length	M+K=80
Guard Interval	K=16
Antenna Configuration	8 Elements Circular Array
Antenna Spacing	Half of Carrier Wavelength
Pulse Shaping Filter	Root-Nyquist
	(role-off factor = 0.5)

The noise power at the equalizer output is given by

$$P_{n} = E \left[\operatorname{tr}[\mathbf{Tn}(n)(\mathbf{Tn}(n))^{H}] \right]$$

= $\sigma^{2} \mathbf{w}^{H} \mathbf{w} \times \operatorname{tr}[\mathbf{D}^{H} \mathbf{\Lambda}^{-1} \mathbf{\Lambda}^{-H} \mathbf{D}]$
= $\sum_{m=1}^{M} \frac{\sigma^{2} \mathbf{w}^{H} \mathbf{w}}{\mathbf{w}^{H} \tilde{\mathbf{R}}_{m} \mathbf{w}}.$ (27)

Therefore, the proposed scheme employs the solution of the minimization problem of

$$\mathbf{w}_{o} = \arg\min_{\mathbf{w}} \sum_{m=1}^{M} \frac{\mathbf{w}^{H} \hat{\mathbf{R}}_{m} \mathbf{w} + \sigma^{2} \mathbf{w}^{H} \mathbf{w}}{\mathbf{w}^{H} \tilde{\mathbf{R}}_{m} \mathbf{w}}$$
(28)

as the array weight.

If we employ the steepest decent method[11] for the adaptive algorithm, the adaptation of the weight vector is given by

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu \sum_{m=1}^{M} \left(\frac{\sigma^2 \mathbf{w}(k) + \hat{\mathbf{R}}_m \mathbf{w}(k)}{\mathbf{w}^H(k) \tilde{\mathbf{R}}_m \mathbf{w}(k)} - \left(\sigma^2 \mathbf{w}^H(k) \mathbf{w}(k) + \mathbf{w}^H(k) \hat{\mathbf{R}}_m \mathbf{w}(k) \right) \\ \times \frac{\tilde{\mathbf{R}}_m \mathbf{w}(k)}{(\mathbf{w}^H(k) \tilde{\mathbf{R}}_m \mathbf{w}(k))^2} \right),$$
(29)

where $\mathbf{w}(k)$ and μ denote the weight vector at the kth iteration and the step-size parameter, respectively.

V. COMPUTER SIMULATION

A. Simulation Condition

Computer simulations are carried out for the following four weights control methods;

- proposed weight control method (Proposed),
- weight control method, which maximizes the SNR at array output (MSA),
- weight control method, which utilizes filtered reference signal (FRS),
- conventional weight control method, which captures only the primary wave (Conventional).

The MSA is proposed for the OFDM adaptive array [8] and the array weights are obtained as an eigenvector corresponding to the maximum eigenvalue of the correlation matrix of the received signal. The FRS is originally proposed for co-channel interference cancellation in a spatial and temporal equalization system[9] and is applied to the OFDM array in [7]. The FRS can receive the delayed waves only within the GI by generating



Fig. 2. Antenna beam patterns: without a path beyond GI

the reference signal with a transversal filter, which has the same filter length as the channel order.

System parameters used in the simulations are summarized in Table I. The transmitted signal block (80 symbols length) consists of the information signal block of 64 symbols length and the CP of 16 symbols length. The first two blocks are used for the transmission of the pilot signal, which is utilized to estimate the channel response and the calculation of the weights. The subsequent 10 received information bearing signal blocks are demodulated using the weights.

Table II shows the channel models used for the evaluation of the SNR and the BER performance. In all the channel models, the delay time of each delayed signal within the GI follows a uniform distribution inside the GI, whereas the distribution of the directions of arrival (DoA) of each path is assumed to be a uniform distribution over a 360[deg] range. The SNR and the BER are evaluated by the average of 100,000 times simulation trials.

B. Antenna Beam Patterns

Fig. 2 shows examples of antenna beam patterns of the weights control methods, where the number of incoming waves is 3 and the DoAs are fixed to be 0, 60, -80[deg], respectively. All the delayed waves are within the GI. In the figure, the conventional method forms nulls toward the delayed waves, while the other methods give gain not only to the primary wave but also to the delayed waves. This means that that the proposed method, MSA and FRS can receive multiple incoming waves, which have different delay times.

On the other hand, Fig. 3 shows antenna beam patterns, where two delayed waves within the GI are incoming from 0 and 60 [deg], respectively, and one delayed wave beyond



Fig. 3. Antenna beam patterns: with a path beyond GI



Fig. 4. E_s/N_0 at the array output (model A)

the GI is in the direction of -80[deg]. Since the MSA can not suppress the delayed wave beyond the GI, here after, the performance of the MSA will not be evaluated when there exists a delayed wave beyond the GI. The antenna beam pattern of the conventional method in Fig. 3 is almost the same as the pattern in Fig. 2. This is because the performance of the conventional method does not depend on the delay times of incoming signals. The proposed method and the FRS give gain to the delayed wave within the GI, while they form a null toward the delayed wave beyond the GI.

C. SNR at the Array Output and the Equalizer Output

Figs. 4 and 5 show the ratio of the average energy per symbol to the noise power density (E_s/N_0) at the array output versus the E_s/N_0 at the array input in channel model A and B, respectively. In both of the channel models, the MSA can achieve the highest E_s/N_0 at the array output. From the figure, we can see that the proposed system, the MSA, and the FRS can increase the E_s/N_0 at the array output due to the multiple delayed signals reception.

Figs. 6 and 7 show the E_s/N_0 at the equalizer output versus the E_s/N_0 at the array input in channel model A and B,



Fig. 5. E_s/N_0 at the array output (model B)



Fig. 6. E_s/N_0 at the equalizer output (model A)

respectively. Now, the proposed system achieves the highest E_s/N_0 at the *equalizer output* in both of the channel models. Although the MSA and the FRS can capture not only the primary wave but also the delayed waves, they can not increase the E_s/N_0 at the equalizer output so much. This is because the MSA and the FRS do not care about the noise enhancement at the equalizer.

D. BER Performance

Figs. 8-11 show the BER performance versus the average received energy per bit to the noise power density (E_b/N_0) per antenna element in channel model A, B, C, and D. The MSA and the FRS can improve the BER performance of the SC-CP system compared as the conventional method, however, the proposed method can achieve the best BER performance among the four weight control methods in all the channel models.

VI. CONCLUSION

We have proposed a new weight control method for the pre-FFT type SC-CP adaptive antenna array based on the maximization of the average SINR at the equalizer output. Since the BER of the SC-CP system depends on the average



Fig. 7. E_s/N_0 at the equalizer output (model B)



Fig. 8. BER performance (model A)

SNR at the equalizer output, the proposed scheme can achieve the optimum weights when there exists no delayed waves beyond the GI. Moreover, the proposed method can suppress the delayed waves beyond the GI, and hence, the ISI and IBI component after the discrete frequency domain equalizer. The antenna beam patterns, E_s/N_0 at the equalizer output, and the BER performance of the proposed method are evaluated via the computer simulations, comparing with the performances of the other weight control methods, such as the MSA, the FRS, and the conventional method. From all the results, it can be concluded that the proposed weights control method can achieve the best performance among the four methods, and the weight control methods for the OFDM adaptive array do not necessarily improve the performance of the SC-CP system, although they can receive not only the primary wave but also the delayed waves.

We are currently investigating the extension of the proposed method to the single carrier block transmission with zeropadding system.



Fig. 9. BER performance (model B)



Fig. 10. BER performance (model C)

REFERENCES

- S. Hara and P. Prasad, "Overview of multicarrier CDMA", IEEE Commun. Mag., vol.35, pp. 126-133, Dec. 1997.
- [2] H. Sari, G. Karam and I. Jeanclaude, "Transmission Techniques for Digital Terrestrial TV Broadcasting", *IEEE Commun. Mag.*, vol.33, pp. 100-109, Feb. 1995.
- [3] D. Falconer, S. L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency Domain Equalization for Single-Carrier Broadband Wireless System", *IEEE Commun. Mag.*, vol.40, pp. 58-66, Apr. 2002.
- [4] Z. Wang and G. B. Giannakis, "Wireless Multicarrier Communications", *IEEE Signal Processing Mag.*, vol.17, pp.29-48, May 2000.
- [5] K. Hayashi, T. Kojima, and H. Sakai, "An Adaptive Antenna Array for Single Carrier Modulation System with Cyclic Prefix", Proc. of VTC2003 Spring, Jeju, Korea, Apr. 2003.
- [6] Y.-P. Lin and S.-M. Phong, "Analytic BER Comparison of OFDM and Single Carrier Systems", *Proc. of SMMSP'02*, Toulouse, France, pp. 127-130, Sep. 2002,
- [7] T. Matsue, T. Hattori, S. Fukui, T. Ina, and N. Kikuma, "A study on adaptive array antenna for OFDM", *Proc. of IEICE Conference* (in Japanese), B-1-83, 2001.
- [8] S. Hane, Y. Hara, and S. Hara, "Selective Signal Reception for OFDM Adaptive Array Antenna", *Technical report of IEICE* (in Japanese), A·P2001-69, pp. 35-41, Aug. 2001.
- [9] M.-L. Leou and C.-C. Yeh, "Novel Hybrid of Adaptive Array and



Fig. 11. BER performance (model D)

Equalizer for Mobile Communications", *IEEE Trans. Veh. Technol.*, vol.49, pp. 1-10, Jan. 2000.

- [10] G. H. Golub and C. F. van Loan, *Matrix Computations*, 3rd ed. Baltimore, MD, Johns Hopkins Univ. Press, 1996.
- [11] B. Farhang-Boroujeny, Adaptive Filters Theory and Applications, Chichester, John Wiley & Sons, 1998.
- [12] A. V. Oppenheim and R. W. Schafer, *Discrete-Time Signal Processing*, Englewood Cliffs, NJ, Prentice Hall, Inc., 1989.