ON THE EFFECT OF TRANSMITTER IQ IMBALANCE AT OFDMA RECEIVERS

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ABSTRACT

One limiting issue in implementing high-speed wireless systems is the impairment associated with analog processing due to component imperfections. In uplink transmission of multiuser systems, a major source of such impairment is IQ imbalance (IQI) introduced at multiple transmitters. In this paper, we analyze the effect of the transmitter (Tx) IQI on OFDMA receiver. To cope with the inter-user interference problem due to Tx IQI, we propose a novel subcarrier allocation scheme, which has high tolerance to such Tx IQ distortion.

***Index Terms*—** IQ Imbalance, OFDMA, WiMAX

1. INTRODUCTION

Recently, orthogonal frequency division multiplexing (OFDM) [1] based multiple access schemes are attracting much attention for uplink physical layer protocols in high-speed wireless networks. The most prominent example is orthogonal frequency division multiple access (OFDMA) employed in IEEE802.16e (WiMAX) [2].

A low-cost implementation of such physical layers by using direct conversion (or Zero IF) architecture [**?**] is desirable in view of mass deployment, but challenging due to impairments associated with the analog components, e.g., the In-phase/Quadrature-phase imbalance (IQI) and carrier frequency offset [4]. In general, it is difficult to efficiently and entirely eliminate such analog impairments in the analog domain due to power consumption, size and cost of the devices. Therefore, efficient compensation techniques in the digital baseband domain are needed for the transceivers.

In this paper, we focus on the transmitter (Tx) IQI and investigate the effect at the OFDMA receiver. The IQI is the mismatch between I and Q balances in the IQ mod/demodulation of the complex-valued signals at the transceivers. Especially, in the mobile uplink, the IQIs introduced at the multiple user handsets might be significant performance limiting issue. From our analysis, we show that the Tx IQIs cause inter-user interference (IUI) at the receiver and the appearance of such IUI uniquely determined by the employed subcarrier allocation. What is more, by deriving the signal-to-interference ratio (SIR) after user separation, we suggest that the IUI possibly cause the serious and unfair performance degradation on some specific users in frequency selective environments. Heretofore the analysis on the impact of Tx and/or receiver IQIs and some compensation methods have been reported for the OFDM receivers [5–7]. However, to the best of our knowledge, few works has been done focusing on the such unfair IUI problem due to the Tx IQIs in OFDMA systems.

To cope with the IUI problem, we propose a novel subcarrier allocation method, which has high tolerance to the Tx IQIs. Recalling, the subcarrier allocation dependent IUI is the main source of performance deterioration due to the Tx IQIs. Hence, it is possible to avoid the IUI and efficiently suppress the ill effect of the Tx IQIs at the OFDMA receiver by employing some specific subcarrier allocations. The validity of our analysis and the efficiency of the proposed method will be evaluated via computer simulations

The notations used in this paper are as follows: the bold capital letters are used to denote column vectors or matrices and $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^{\mathrm{H}}$ are complex conjugate, transpose, and Hermitian transpose of $(\cdot)$ respectively. An $M \times M$ identity matrix is denoted by $\mathbf{I}_M$ and $\mathbf{0}_{M\times N}$ is an $M \times N$ all-zero matrix. We use diag$\{\cdot\}$ as a diagonal matrix, $\|\cdot\|$ denotes the Euclidean norm, and $\mathrm{E}[\cdot]$ is the expectation operation. Let a $KM \times KM$ matrix $\mathbf{W}$ represents KM-point discrete Fourier transform (DFT) matrix, whose (m, n) element is $\mathbf{W}(m,n) = \frac{1}{\sqrt{KM}} e^{-j\cdot 2\pi \frac{(m-1)(n-1)}{KM}}$. On the other hand, a $M \times M$ matrix $\bar{\mathbf{W}}$ denotes M-point DFT matrix.

2. OFDMA SIGNAL MODEL WITH IQ IMBALANCE

We start with describing the received signal model for OFDMA with general subcarrier allocation in the presence of Tx IQI. Then the impact of the Tx IQIs at the conventional receiver is investigated by obtaining the SIR. Furthermore, we specify the models to the systems with two major subcarrier allocations, namely interleaved and localized, and show the unfair IUIs depend on employed subcarrier allocations.

2.1. OFDMA with General Subcarrier Allocation

In the followings, we assume a system with K users and the available KM subcarriers are divided into K subcarrier groups equally for simplicity. Let $\mathbf{s}_k = [s_k(0), \cdots, s_k(M-1)]^{\mathrm{T}}$ designate a block of M data symbols transmitted by a user with index k ($k = 1, 2, \cdots, K$). The data symbols may result from application of a modulation scheme like QAM to either forward error control coded or to uncoded data bits. The assignment of the data symbols $\mathbf{s}_k$ to the user specific set of M subcarriers can be represented by $KM \times M$ mapping matrix $\mathbf{M}_k$ and a KM-point IDFT matrix $\mathbf{W}^{\mathrm{H}}$. The mapping matrix $\mathbf{M}_k$ has a unique element of one in each column and has properties of

$$\mathbf{M}_l^{\mathrm{T}}\mathbf{M}_k = \begin{cases} \mathbf{0}_{M\times M} & (l \neq k) \\ \mathbf{I}_M & (l = k) \end{cases}. \tag{1}$$

The transmitted signal of the k th user is given by

$$\mathbf{x}_k = \mathbf{W}^{\mathrm{H}}\mathbf{M}_k\mathbf{s}_k. \tag{2}$$

Practically, the baseband signal $\mathbf{x}_k$ is up-converted to the radio frequency signal by using a local oscillator (LO) of the carrier frequency before transmission. Ideally, the LO outputs for the I and Q branches should have equal amplitudes and phase difference of $\pi/2$. However, in practice, the matching of I and Q signals is habitually imperfect and this leads the amplitude and phase imbalance between the I and Q signals. Such impairments are known as Tx IQI and this severely limits the performance of the receiver, especially if cheap components or architectures, e.g., direct conversion architecture [3], are employed. From [4], the IQ distorted transmitted signal can be modeled as

$$\tilde{\mathbf{x}}_k = \alpha_k \mathbf{x}_k + \beta_k \mathbf{x}_k^* = \alpha_k \mathbf{W}^{\mathrm{H}} \mathbf{M}_k \mathbf{s}_k + \beta_k \mathbf{W} \mathbf{M}_k \mathbf{s}_k^*, \quad (3)$$

where two complex scalers α_k and β_k are given by $\alpha_k = \cos\theta_k + j \cdot \epsilon_k \sin\theta_k$, $\beta_k = \epsilon_k \cos\theta_k - j \cdot \sin\theta_k$ where θ_k and ϵ_k denote the phase and amplitude imbalances between the I and Q branches of the transmitted signal of the k th user, respectively. Generally, the degree of the imbalance is evaluated by using the image rejection ratio (IRR) , for the k th user's transmitted signal, which is defined by $\mathrm{IRR}_k = \mathrm{E}[\|\alpha_k \mathbf{x}_k\|^2]/\mathrm{E}[\|\beta_k \mathbf{x}_k^*\|^2] = |\alpha_k|^2/|\beta_k|^2$.

The IQ distorted signals from K users are then received at the base station after passing through the frequency selective channel. It is well known that, by the use of cyclic prefix (CP) as guard intervals, the selective fading channel is converted into parallel subchannles in discrete frequency domain. Let a $KM \times KM$ diagonal matrix $\mathbf{\Lambda}_k$ denotes the discrete frequency response of the channel of the k th user, the OFDMA received signal after CP removal can be described as

$$\mathbf{r} = \mathbf{W}^{\mathrm{H}} \left(\sum_{k=1}^{K} \alpha_k \mathbf{\Lambda}_k \mathbf{M}_k \mathbf{s}_k + \beta_k \mathbf{\Lambda}_k \mathbf{W}^2 \mathbf{M}_k \mathbf{s}_k^* \right) + \mathbf{n}, \quad (4)$$

where the $KM \times 1$ vector $\mathbf{n}$ denotes additive noise. It should be mentioned that, in this paper, we mainly focuses on the IUI problem due to the Tx IQ imbalance in selective fading environments, which has not given much attention so far. Thus, for simplicity, we only deal with the Tx IQ distortion and assume that some other analog impairments, such as Rx IQI and CFO, are negligibly small or has a priori compensated with some advanced signal processing techniques such as [5, 7].

Next, we describe the per-user received signal after multi-user detection. Here, we assume that the user separation is performed in the conventional manner, i.e., the received signal component on the set of subcarriers assigned to a user is extracted out by using the corresponding allocation matrix in discrete frequency domain. The received signal of the l th user ($l = 1, 2, \cdots, K$) is given by

$$\begin{aligned} \mathbf{r}_l &= \mathbf{M}_l^{\mathrm{T}} \mathbf{W} \mathbf{r} \\ &= \alpha_l \mathbf{\Lambda}_{ll} \mathbf{s}_l + \left(\sum_{k=1}^{K} \beta_k \mathbf{\Lambda}_{lk} \mathbf{M}_l^{\mathrm{T}} \mathbf{W}^2 \mathbf{M}_k \mathbf{s}_k^* \right) + \mathbf{n}_l, \end{aligned} \quad (5)$$

where $\mathbf{\Lambda}_{lk} = \mathbf{M}_l^{\mathrm{T}} \mathbf{\Lambda}_k$, and $\mathbf{n}_l = \mathbf{M}_l^{\mathrm{T}} \mathbf{W} \mathbf{n}$. The second term in the right-hand side of (5) represents the interference due to the Tx IQIs. Since $\mathbf{W}^2$ is a kind of permutation matrix, $\mathbf{M}_l \mathbf{W}^2 \mathbf{M}_k$ consist of 0 or 1 elements and hence the appearance of the interference largely depend on the employed subcarrier mappings. In order to clarify the impact of the interference on the performance of OFDMA receivers, we derive the per-subcarrier SIRs for each user. By assuming,

$$\mathrm{E}[\mathbf{s}_k \mathbf{s}_l^*] = \begin{cases} \sigma_s^2 \mathbf{I}_M & (k = l) \\ \mathbf{0}_{M \times M} & (\text{otherwise}) \end{cases}, \quad (6)$$

the SIR of the m th subcarrier ($m = 1, \cdots, M$) in the l th user's received signal is given by

$$\mathrm{SIR}_l(m) = \frac{|\alpha_l|^2}{|\beta_l|^2 \mathbf{m}_{ll}(m) + \sum_{k=1\ (k \neq l)}^{K} |\beta_k|^2 \frac{|\mathbf{\Lambda}_{lk}(m)|^2}{|\mathbf{\Lambda}_{ll}(m)|^2} \mathbf{m}_{lk}(m)}, \quad (7)$$

where the $M \times 1$ vector $\mathbf{m}$ is diag$\{\mathbf{m}_{lk}\}= \mathbf{M}_l^{\mathrm{T}} \mathbf{W}^2 \mathbf{M}_k \mathbf{M}_k^{\mathrm{T}}$ and $\mathbf{\Lambda}_{lk}(m)$ denotes the m th diagonal element of $\mathbf{\Lambda}_{lk}$. In the denominator of (7), the first term corresponds to the interference due to the l th user's own Tx IQI. Meanwhile, the second term comes from the interference caused by other users' IQIs, namely the IUI. In the OFDM system, or equivalently $K = 1$ and the mapping matrix $\mathbf{M}_1 = \mathbf{I}_M$ in our formulations, the SIR is geiven by $\mathrm{SIR}_1(m) = |\alpha_1|^2/|\beta_1|^2$. Obviously, there is no IUI problem in OFDM system and the $\mathrm{SIR}_1(m)$ is the same as the IRR_l. Therefore high IRR level at the transmitter's analog front-end directly means low interference power at the receiver. Meanwhile, in the OFDMA system, the received signal suffer from the IUI depending on the employed subcarrier allocation and the resulting $\mathrm{SIR}_l(m)$ is far from the IRR_l. Furthermore, since the impact of such IUI is proportional to the channel gains $|\mathbf{\Lambda}_{lk}(m)|^2/|\mathbf{\Lambda}_{ll}(m)|^2$, $\mathrm{SIR}_l(m)$ possibly comes down to the serious level in selective fading environments.

Next, we specify the signal models for OFDMA systems with interleaved and localized allocations and also derive the SIR for each allocation.

2.2. Interleaved OFDMA

Interleaved OFDMA, also known as distributed OFDMA has the user dependent comb shape subcarrier allocation. Interleaved mapping matrix for the k th user is given by

$$\mathbf{M}_k = \mathbf{\Pi}^{k-1} \mathbf{U}, \quad (8)$$

where $\mathbf{U}$ is defined as $KM \times M$ expander matrix, whose Km-th row is equal to the m-th of $\mathbf{I}_M$ and the other rows are zero vectors, and a $KM \times KM$ circular down-shifting matrix $\mathbf{\Pi}$ whose (m, n) element is given by

$$\mathbf{\Pi}(m, n) = \begin{cases} 1 & (m = n + 1 \mod KM) \\ 0 & (\text{otherwise}) \end{cases}. \quad (9)$$

After some algebra (see [8] for details), we can specify (5) for the interleaved OFDMA as

$$\mathbf{r}_l = \alpha_l \mathbf{\Lambda}_{ll} \mathbf{s}_l + \beta_{f(l)} \mathbf{\Lambda}_{lf(l)} \bar{\mathbf{\Pi}}^{-g(l)} \bar{\mathbf{W}}^2 \mathbf{s}_{f(l)}^* + \mathbf{n}_l, \quad (10)$$

where $f(l)$ and $g(l)$ are given by

$$\{f(l), g(l)\} = \begin{cases} \{1, 0\} & (l = 1) \\ \{K - l + 2, 1\} & (\text{otherwise}) \end{cases}, \quad (11)$$

and $\bar{\mathbf{\Pi}}$ denotes a $M \times M$ circular down-shifting matrix. For example, when $K = 4$, the relation ship between l and $f(l)$ is given as follows,

l	1	2	3	4
$f(l)$	1	4	3	2

Under the same assumption in (6), the per-subcarrier received SIR of the l th user is given by

$$\mathrm{SIR}_l(m) = \frac{|\alpha_l \mathbf{\Lambda}_{ll}(m)|^2}{|\beta_{f(l)} \mathbf{\Lambda}_{lf(l)}(m)|^2}, \quad (12)$$

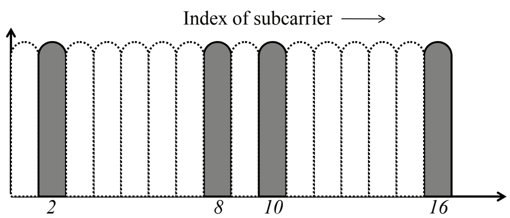

Fig. 1. Subcarrier allocation of the 2nd user in the proposed IUI free OFDMA where $K = 4$ and $M = 4$

where $m = 1, \cdots, M$. Interestingly, in the interleaved OFDMA, the l th user's received signal interfered only by the $f(l)$ th user's image component. In particular, when $l \neq f(l)$, the interference can be considered as an IUI and, with respect to user fairness, such IUI problem is quite undesirable since the impairment in IQ modulation of a user terminal make trouble not on its own performance but on the other's. On the other hand, the 1 st and the $(K/2+1)$ th users successfully avoid the IUI and the resulting SIR is equivalent to the IRR. We evaluate this unique phenomenon later in our computer simulations.

2.3. Localized OFDMA

The localized OFDMA has block-wise subcarrier allocation, and the mapping matrix for the k th user is given by

$$\mathbf{M}_k = \mathbf{\Pi}^{(k-1)M}\mathbf{V}, \tag{13}$$

where $KM \times M$ matrix $\mathbf{V} = \left[\mathbf{I}_M \mathbf{0}_{M\times(K-1)M}\right]^{\mathrm{T}}$. As to the per-user received signal of the localized OFDMA, we can obtain $\mathbf{r}_l$ as

$$\begin{aligned}\mathbf{r}_l &= \alpha_l \mathbf{\Lambda}_{ll}\mathbf{s}_l + \beta_{f(l)}\mathbf{\Lambda}_{lf(l)}\mathbf{J}\mathbf{s}^*_{f(l)} \\ &\quad + \beta_{e(l)}\mathbf{\Lambda}_{le(l)}(\bar{\mathbf{W}}^2 - \mathbf{J})\mathbf{s}^*_{e(l)} + \mathbf{n}_l, ,\end{aligned} \tag{14}$$

where $e(l) = K - l + 1$ and the $M \times M$ matrix $\mathbf{J}$ is defined as $\mathbf{J}(1,1) = 1$ and the other elements are 0. For example, when $K = 4$,

$$\begin{array}{c|cccc} l & 1 & 2 & 3 & 4 \\ \hline e(l) & 4 & 3 & 2 & 1 \end{array}.$$

The resulting per-subcarrier SIR of the l th user is given by

$$\mathrm{SIR}_l(m) = \begin{cases} \frac{|\alpha_l \mathbf{\Lambda}_{ll}(m)|^2}{|\beta_{f(l)}\mathbf{\Lambda}_{lf(l)}(m)|^2} & (m = 1) \\ \frac{|\alpha_l \mathbf{\Lambda}_{ll}(m)|^2}{|\beta_{e(l)}\mathbf{\Lambda}_{le(l)}(m)|^2} & (m > 1) \end{cases}, \tag{15}$$

where $m = 1, \cdots, M$. In the localized OFDMA, the interference on the l th user received signal consist of the IUIs from $f(l)$ and $e(l)$ th user's image components. Actually, from (15), the IUI from the $e(l)$ th user is the main source of the performance degradation when M is sufficiently large.

3. IUI FREE SUBCARRIER ALLOCATION

Here, we propose a novel subcarrier mapping method which has high tolerance to the Tx IQ distortion. Recalling that the IUI component is the major source of the performance degradation due to the Tx IQI and also such IUI emerges depending on the employed subcarrier allocation $\mathbf{M}_k$. Therefore, it is possible to choose a specific set of subcarrier assignments in which all the users can avoid the IUI due to their Tx IQIs. Clearly, from (5), if

$$\mathbf{M}_l^{\mathrm{T}}\mathbf{W}^2\mathbf{M}_k = \mathbf{0}_{M\times M}(k \neq l). \tag{16}$$

holds, there is no IUI. Actually, there are many choices for a set of $\mathbf{M}_k$ $(k = 1, \cdots, K)$ which holds (16) and (1). One simple solution is to assign certain two interleaved allocations for one user based on (10): First we temporarily consider the system with $K' = 2K$ users where each user occupies $M/2$ subcarriers and design $2K$ interleaved mapping matrices $\mathbf{M}'_{k'}$ $(k' = 1, \cdots, 2K)$ (Here we assume K and M are power of 2 for simplicity). The resulting IUI only appears between certain users k' and $K' - k' + 2$ as in (11). Therefore, in the system with K users, we can derive a set of IUI free subcarrier mapping matrices as

$$\mathbf{M}_k = \begin{cases} \left[\mathbf{M}'_1 \;\; \mathbf{M}'_{K+1}\right] & (k = 1) \\ \left[\mathbf{M}'_k \;\; \mathbf{M}'_{2K-k+2}\right] & (k \neq 1) \end{cases}, \tag{17}$$

where $k = 1, \cdots, K$. An example of subcarrier allocation in the proposed IUI free OFDMA is illustrated in Fig. 1 where $K = 4$ and $M = 4$.

By the use of such IUI free subcarrier allocations, we can efficiently avoid the performance deterioration due to Tx IQIs without knowing the imbalance parameters and channel coefficients a priori.

4. NUMERICAL RESULTS

In our simulations, we have employed uncoded 16QAM modulation scheme, and consider the system $K = 4$ users, where all the users are assigned the same transmit power. The number of subcarriers per-user $M = 128$ and the length of CP is set to be 32. For the channel, we considered Rayleigh fading channels of length 10. At the receiver, the frequency domain zero-forcing equalizer with perfect channel state information is employed and the user separation is conducted in the conventoinal manner as in (5).

Firstly, we consider the situation where only the 1st and 2nd users' transmitter suffer from IQI where the imbalance parameters are randomly generated and resulting IRR_1 and IRR_2 are about 35 dB. Fig. 2 shows the user-by-user BER performance of the interleaved OFDMA versus the trasnmitted SNR. From the figure, we can see the significant performance deterioration only on the 4th user. As we have shown in Sec 2.2, the IUI problem occuers only between the 2nd and 4th users in this scenario. Therefore, the resulting $\mathrm{SIR}_4(m)$ possibly become fatal according to the channel conditions, while $\mathrm{SIR}_1(m) = \mathrm{IRR}_1$ and the interference of 35 dB slightly degrade the performance of the 1st user. Fig. 3 represents the performance of the localized OFDMA in the same scenario. In this case, the most of IUI due to the Tx IQIs of the 1st and 2nd users appears on the 4th and 3rd users respectively. Therefore we can see the significant performance degradation on the 3rd and 4th users. These results clearly verify the validity of our analysis.

Next, we show the efficiency of the proposed IUI-free subcarrier allocation over the random allocation. Fig. 4 show the performances of the OFDMA with the proposed and randomly generated subcarrier mappings where all the transmitters suffer from IQI of IRR $\approx$ 35dB and other settings are the same as in Fig. 2. In the randomly allocated OFDMA, the IUI possibly occurs between all the users and the BER performances are seriously degraded. Meanwhile, the OFDMA with proposed allocation efficiently avoid the IUI problem and significantly improve the BER performance.

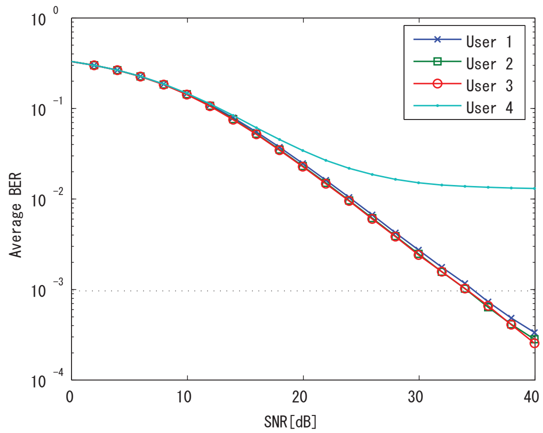

Fig. 2. The BER performance of the interleaved OFDMA with the conventional receiver

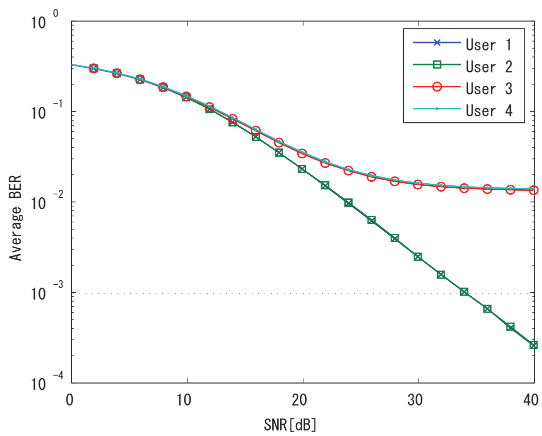

Fig. 3. The BER performance of the localized OFDMA

5. CONCLUSION

In this paper, the effect of the Tx IQIs at the OFDMA receiver is investigated. We reveal the IUI due to the IQI is the major source of performance degradation and such IUI emerges depend on the subcarrier mapping employed in the system. A novel subcarrier allocation method is proposed to cope with the severe IUI problem. Under Tx IQIs, we can significantly improve the BER performance of the OFDMA receivers by just employing the proposed mapping.

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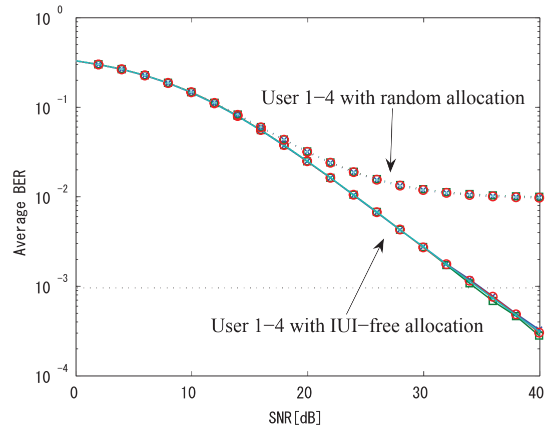

Fig. 4. The BER performance of the OFDMA with the random and IUI-free subcarrier allocation

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