PAPER

Per-Tone Equalization for Single Carrier Block Transmission with Insufficient Cyclic Prefix*

Kazunori HAYASHI† *and* **Hideaki SAKAI**†, ***Members***

SUMMARY This paper proposes per-tone equalization methods for single carrier block transmission with cyclic prefix (SC-CP) systems. Minimum mean-square-error (MMSE) based optimum weights of the per-tone equalizers are derived for SISO (single-input single-output), SIMO (single-input multiple-output), and MIMO (multiple-input multiple-output) SC-CP systems. Unlike conventional frequency domain equalization methods, where discrete Fourier transform (DFT) is employed, the per-tone equalizers utilize sliding DFT, which makes it possible to achieve good performance even when the length of the guard interval is shorter than the channel order.

Computer simulation results show that the proposed equalizers can significantly improve the bit error rate (BER) performance of the SISO, SIMO, and MIMO SC-CP systems with the insufficient guard interval.

key words: *per-tone equalization, single carrier block transmission, cyclic prefix*

1. Introduction

Cyclic prefix based block transmission schemes, such as orthogonal frequency division multiplexing (OFDM)[1],[2], discrete multitone (DMT)[3], and single carrier block transmission with cyclic prefix (SC-CP)[4],[5], have been drawing much attention as promising candidates for broadband communications systems. The insertion of the cyclic prefix as a guard interval at the transmitter and the removal of the cyclic prefix at the receiver eliminates inter-block interference (IBI) due to multipath fading. Moreover, the insertion and the removal of the cyclic prefix converts the effect of the channel from a linear convolution into a circular convolution, therefore, the inter-symbol interference (ISI) of the received signal can be equalized by a simple frequency domain equalizer (FDE) using fast Fourier transform (FFT). However, the existence of delayed signals beyond the guard interval causes residual ISI and IBI after the FDE, and hence deteriorates the performance of the block transmission with the cyclic prefix. In order to overcome the performance degradation of the OFDM or the DMT systems due to the insufficient guard interval, a considerable number of studies have been made on the issue, such as introduction of a temporal equalizer[6]- [8], and per-tone equalization[9],[10]. Although the temporal equalizer can mitigate the IBI and the ISI due to the delayed signals, the per-tone equalizer outperforms the temporal equalizer. This is because the per-tone equalizer can use optimum weights at each tone, while a common set of weights have to be used for all the tone in the temporal equalization approach. Furthermore, the per-tone equalizer can be efficiently implemented by using sliding discrete Fourier transform (DFT)[11].

In this paper, we propose per-tone equalization methods for SISO (single-input single-output), SIMO (single-input multiple-output) and MIMO (multiple-input multiple-output) SC-CP systems and derive minimum mean-square-error (MMSE) based optimum weights of the per-tone equalizers. The proposed methods can be regarded as extensions of [9] and [10] into the SC-CP systems, however, we present more simplified and intuitive descriptions of the per-tone equalizers, using unified data modeling for the SISO, SIMO and MIMO SC-CP systems. Also, computer simulations are conducted in order to evaluate the bit error rate (BER) performance of the proposed per-tone equalizers with the insufficient guard interval.

The paper is organized as follows. Section 2 illustrates configurations and data modeling methods of the SISO, SIMO and MIMO SC-CP systems with the proposed per-tone equalizers. The MMSE optimum equalizer weights are also presented in Section 2. The BER performances of the proposed SC-CP systems are evaluated via Computer simulations in Section 3, and finally, we conclude the paper in Section 4.

2. Proposed Per-Tone Equalization Scheme

The following notations are used for describing the proposed systems. K is the length of the guard interval, N the FFT size, T the number of equalizer taps at each tone, and L is the channel order. An $N \times N$ identity matrix will be denoted as $\mathbf{I}_N$, a zero matrix of size $K \times N$ will be denoted as $\mathbf{0}_{K\times N}$, and a DFT matrix of size $N \times N$, whose (i, j) element is $\frac{1}{\sqrt{N}} e^{-j\frac{2\pi(i-1)(j-1)}{N}}$, as $\mathbf{F}$. We will use $E[\cdot]$ to denote ensemble average, $(\cdot)^T$ for transpose, $(\cdot)^H$ for Hermitian transpose, $tr(\cdot)$ for trace, and $(\cdot)^*$ for complex conjugate.

Manuscript received January 1, 2000.
Manuscript revised January 1, 2000.
Final manuscript received January 1, 2000.

†The authors are with Graduate School of Informatics, Kyoto University, Yoshida Honmachi Sakyo-ku, Kyoto, 606-8501, JAPAN

*This research was supported in part by the International Communications Foundation (ICF), Tokyo.

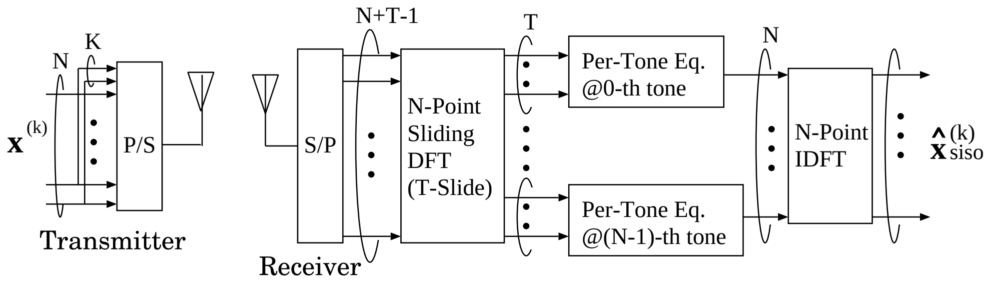

Fig. 1 Configuration of Proposed SISO SC-CP System

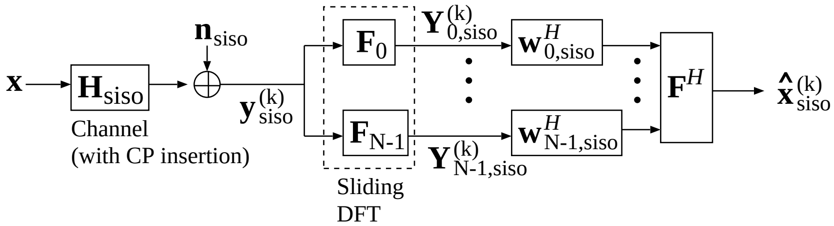

Fig. 2 Schematic Diagram of Proposed SISO SC-CP System

2.1 SISO (Single-Input Single-Output) SC-CP System

Fig.1 shows a configuration of the SISO SC-CP system with the proposed per-tone equalizer. Also, Fig.2 shows the corresponding schematic diagram. Let denote the kth information signal block as $\mathbf{x}^{(k)} = [x_0^{(k)}, \ldots, x_{N-1}^{(k)}]^T$. Since we consider the case where $K < L$ in this paper, the received signal block $\mathbf{y}^{(k)}_{siso}$, which corresponds to the kth transmitted information signal block $\mathbf{x}^{(k)}$, includes not only $\mathbf{x}^{(k)}$ but also $\mathbf{x}^{(k-1)}$ or $\mathbf{x}^{(k+1)}$, due to the IBI caused by the insufficient guard interval. Therefore, we define a transmitted signal vector including three consecutive information signal blocks as

$$\mathbf{x} = [\mathbf{x}^{(k-1)T}\ \mathbf{x}^{(k)T}\ \mathbf{x}^{(k+1)T}]^T. \tag{1}$$

Note that $\mathbf{x}^{(k)T}$ stands for the transposed of the kth transmitted signal vector of $\mathbf{x}^{(k)}$. Also, since the cyclic prefix insertion operation can be denoted as

$$\mathbf{T}_{cp} = \begin{bmatrix} \mathbf{0}_{K\times(N-K)} & \mathbf{I}_K \\ \multicolumn{2}{c}{\mathbf{I}_N} \end{bmatrix}, \tag{2}$$

the cyclic prefix insertion for the three consecutive information signal blocks $\mathbf{x}$ can be written by a block diagonal matrix

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_{cp} & \mathbf{0}_{(N+K)\times N} & \mathbf{0}_{(N+K)\times N} \\ \mathbf{0}_{(N+K)\times N} & \mathbf{T}_{cp} & \mathbf{0}_{(N+K)\times N} \\ \mathbf{0}_{(N+K)\times N} & \mathbf{0}_{(N+K)\times N} & \mathbf{T}_{cp} \end{bmatrix}. \tag{3}$$

The received signal vector $\mathbf{y}^{(k)}_{siso}$ of size $(N+T-1)\times 1$ is given by

$$\mathbf{y}^{(k)}_{siso} = \begin{bmatrix} y^{(k),0}_{siso} \\ \vdots \\ y^{(k),N+T-2}_{siso} \end{bmatrix} = \mathbf{H}_{siso}\mathbf{x} + \mathbf{n}_{siso}, \tag{4}$$

where $\mathbf{n}_{siso}$ is a zero mean white noise vector of covariance matrix $\sigma_n^2\mathbf{I}_{N+T-1}$ and $\mathbf{H}_{siso}$ is a channel matrix including the effect of the cyclic prefix insertion defined by

$$\mathbf{H}_{siso} = \begin{bmatrix} & h_L & \ldots & h_0 & 0 & \ldots & 0 & \\ & 0 & \ddots & & \ddots & \ddots & \vdots & \\ \mathbf{0}_{(1)} & \vdots & \ddots & \ddots & & \ddots & 0 & \mathbf{0}_{(2)} \\ & 0 & \ldots & 0 & h_L & \ldots & h_0 & \end{bmatrix} \mathbf{T}. \tag{5}$$

Here, $\{h_0, \ldots, h_L\}$ denotes a channel impulse response, and $\mathbf{0}_{(1)}$ and $\mathbf{0}_{(2)}$ denotes zero matrices, which have $N+T-1$ rows. The number of columns of $\mathbf{0}_{(1)}$ and $\mathbf{0}_{(2)}$ determines the zero reference delay and are determined so that the energy of $\mathbf{x}^{(k)}$ in $\mathbf{y}^{(k)}_{siso}$ is maximized.

In cyclic prefix based systems, the DFT is commonly utilized for the frequency domain equalization. On the other hand, the sliding DFT plays a crucial role in the per-tone equalization scheme. The sliding DFT algorithm performs N-point DFTs with shifting the sliding window as shown in Fig.3. Since the received signal vector $\mathbf{y}^{(k)}_{siso}$ has $N+T-1$ elements, we can perform the N-point DFT operation T times. This means that we have T data samples in the DFT domain for each tone. The $T\times 1$ vector $\mathbf{Y}^{(k)}_{i,siso}$, which is composed by a sliding DFT output corresponding to the ith tone, can be written as

$$\mathbf{Y}_{i,siso}^{(k)} = \mathbf{F}_i \mathbf{y}_{siso}^{(k)}, \tag{6}$$

where $\mathbf{F}_i$ denotes the sliding DFT operation on the ith tone. Using $W_N = e^{\frac{-j2\pi}{N}}$, $\mathbf{F}_i$ of size $T \times (N+T-1)$ is defined by

$$\mathbf{F}_i = \frac{1}{\sqrt{N}} \cdot \begin{bmatrix} 1 & W_N^i & \dots & W_N^{i(N-1)} & 0 & \dots & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \dots & 0 & 1 & W_N^i & \dots & W_N^{i(N-1)} \end{bmatrix}. \tag{7}$$

Note that, we utilize the matrix $\mathbf{F}_i$ for the sliding DFT operation throughout the paper in order to denote the per-tone equalizer output in a simple form, however, the sliding DFT can be realized much more efficiently by using the circular shift property of the DFT[11].

The per-tone equalizer output signal $\hat{\mathbf{x}}_{siso}^{(k)}$ is given by

$$\hat{\mathbf{x}}_{siso}^{(k)} = \mathbf{F}^H \begin{bmatrix} \mathbf{w}_{0,siso}^H \mathbf{F}_0 \\ \vdots \\ \mathbf{w}_{N-1,siso}^H \mathbf{F}_{N-1} \end{bmatrix} \mathbf{y}_{siso}^{(k)}, \tag{8}$$

where $\mathbf{w}_{i,siso}$ is a $T \times 1$ per-tone equalizer weight vector at the ith tone. The optimum weight $\mathbf{w}_{i,siso}^{opt}$ in the MMSE sense is obtained by solving the following minimization problem.

$$\mathbf{w}_{i,siso}^{opt} = \arg \min_{\mathbf{w}_{i,siso}} J_{siso}, \tag{9}$$

$$J_{siso} = E\left[tr\left\{ \left(\hat{\mathbf{x}}_{siso}^{(k)} - \mathbf{x}^{(k)} \right) \left(\hat{\mathbf{x}}_{siso}^{(k)} - \mathbf{x}^{(k)} \right)^H \right\} \right]. \tag{10}$$

Assuming that the transmitted signals and the channel noise are independent of each other and the covariance matrix of the transmitted signal vector $\mathbf{x}$ is $E[\mathbf{x}\mathbf{x}^H] = \mathbf{I}_{3N}$, the cost function J_{siso} can be calculated as

$$\begin{aligned} J_{siso} =& tr\left\{ \mathbf{F}^H \mathbf{V}_{siso} \mathbf{H}_{siso} \mathbf{H}_{siso}^H \mathbf{V}_{siso}^H \mathbf{F} \right. \\ & + \sigma_n^2 \mathbf{F}^H \mathbf{V}_{siso} \mathbf{V}_{siso}^H \mathbf{F} \\ & \left. - \mathbf{F}^H \mathbf{V}_{siso} \bar{\mathbf{H}}_{siso} - \bar{\mathbf{H}}_{siso}^H \mathbf{V}_{siso}^H \mathbf{F} + \mathbf{I}_N \right\}, \\ =& \sum_{i=0}^{N-1} \left(\mathbf{w}_{i,siso}^H \mathbf{F}_i \mathbf{H}_{siso} \mathbf{H}_{siso}^H \mathbf{F}_i^H \mathbf{w}_{i,siso} \right. \\ & + \sigma_n^2 \mathbf{w}_{i,siso}^H \mathbf{F}_i \mathbf{F}_i^H \mathbf{w}_{i,siso} - \mathbf{w}_{i,siso}^H \mathbf{F}_i \bar{\mathbf{H}}_{siso} \mathbf{f}_i^H \\ & \left. - \mathbf{f}_i \bar{\mathbf{H}}_{siso}^H \mathbf{F}_i^H \mathbf{w}_{i,siso} \right) + N, \end{aligned} \tag{11}$$

where $\mathbf{f}_i$ is the ith row vector of the DFT matrix $\mathbf{F}$ and

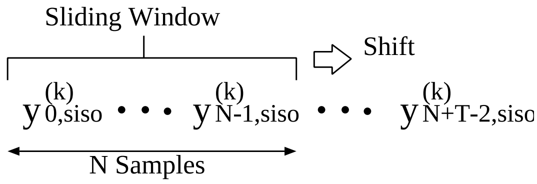

Fig. 3 Sliding DFT

$\bar{\mathbf{H}}_{siso} = \mathbf{H}_{siso} [\mathbf{0}_{N\times N} \;\; \mathbf{I}_N \;\; \mathbf{0}_{N\times N}]^T$ and

$$\mathbf{V}_{siso} = \begin{bmatrix} \mathbf{w}_{0,siso}^H \mathbf{F}_0 \\ \vdots \\ \mathbf{w}_{N-1,siso}^H \mathbf{F}_{N-1} \end{bmatrix}.$$

The differentiation of J_{siso} with respect to the per-tone equalizer weight at the ith tone $\mathbf{w}_{i,siso}^H$ is given by

$$\begin{aligned} \frac{\partial J_{siso}}{\partial \mathbf{w}_{i,siso}^H} =& \mathbf{F}_i \mathbf{H}_{siso} \mathbf{H}_{siso}^H \mathbf{F}_i^H \mathbf{w}_{i,siso} \\ & + \sigma_n^2 \mathbf{F}_i \mathbf{F}_i^H \mathbf{w}_{i,siso} - \mathbf{F}_i \bar{\mathbf{H}}_{siso} \mathbf{f}_i^H, \end{aligned} \tag{12}$$

therefore, the optimum equalizer weight of the ith tone is obtained as

$$\mathbf{w}_{i,siso}^{opt} = \left[\mathbf{F}_i \mathbf{H}_{siso} \mathbf{H}_{siso}^H \mathbf{F}_i^H + \sigma_n^2 \mathbf{F}_i \mathbf{F}_i^H \right]^{-1} \mathbf{F}_i \bar{\mathbf{H}}_{siso} \mathbf{f}_i^H. \tag{13}$$

Note that in the case of $T = 1$ and $L \le K$, the proposed weight of the SISO equalizer perfectly coincides with that of the conventional MMSE based one-tap frequency domain equalizer (FDE) [5] (see Appendix A). Therefore, the proposed equalizer includes the conventional equalizer as a special case.

2.2 SIMO (Single-Input Multiple-Output) SC-CP System

Fig.4 shows a configuration of the proposed SIMO SC-CP system with the per-tone equalizer. In the SIMO system, the transmitter structure, and hence the transmitted signal, is the same as that of the SISO SC-CP system. Also, Fig.5 illustrates the schematic diagram of the proposed SIMO SC-CP system.

Let $\mathbf{y}_{j,simo}^{(k)} = [y_{j,simo}^{(k),0}, \dots, y_{j,simo}^{(k),N+T-2}]^T$ denote the received signal vector at the jth reception antenna. By stacking the vectors $\mathbf{y}_{j,simo}^{(k)}$, $j = 0, \dots, N_r - 1$, we define the received signal vector of size $(N+T-1)N_r \times 1$ for the SIMO SC-CP system as $\mathbf{y}_{simo}^{(k)} = [\mathbf{y}_{0,simo}^{(k)T}, \dots, \mathbf{y}_{N_r-1,simo}^{(k)T}]^T$. The received signal vector $\mathbf{y}_{simo}^{(k)}$ can be written as

$$\mathbf{y}_{simo}^{(k)} = \mathbf{H}_{simo} \mathbf{x} + \mathbf{n}_{simo}, \tag{14}$$

where $\mathbf{n}_{simo}$ is the channel noise vector of size $(N+T-1)N_r \times 1$ and $\mathbf{H}_{simo}$ is the channel matrix defined by

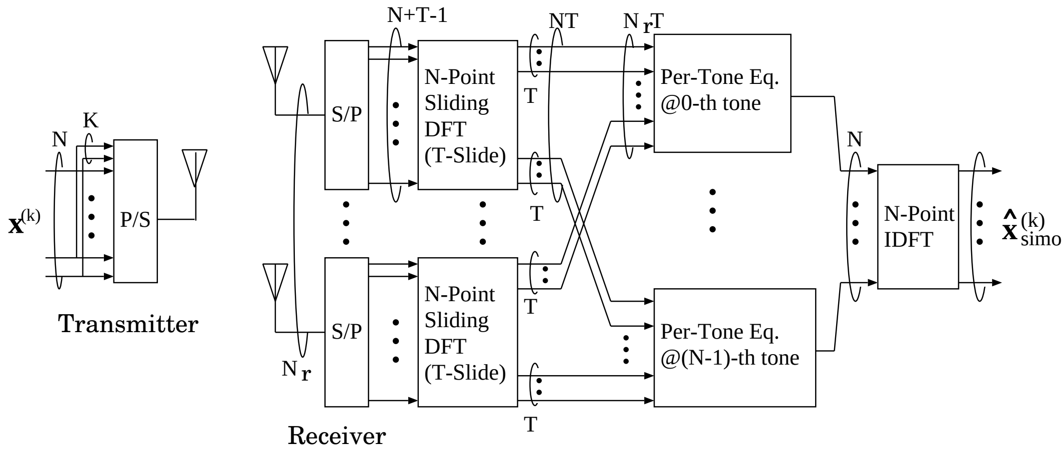

Fig. 4 Configuration of Proposed SIMO SC-CP System

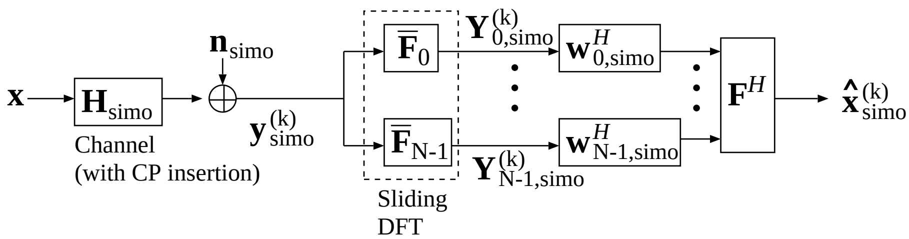

Fig. 5 Schematic Diagram of Proposed SIMO SC-CP System

$$\mathbf{H}_{simo} = \left[\begin{array}{ccccccc} & h_L^0 & \dots & h_0^0 & & \mathbf{0} & \\ & & \ddots & & \ddots & & \\ & \mathbf{0} & & h_L^0 & \dots & h_0^0 & \\ \mathbf{0}_{(3)} & & \vdots & & \vdots & & \mathbf{0}_{(4)} \\ & h_L^{N_r-1} & \dots & h_0^{N_r-1} & & \mathbf{0} & \\ & & \ddots & & \ddots & & \\ & \mathbf{0} & & h_L^{N_r-1} & \dots & h_0^{N_r-1} & \end{array}\right]\mathbf{T}. \tag{15}$$

$\{h_0^j, \dots, h_L^j\}$ denotes a channel impulse response between the transmission antenna and the jth reception antenna. $\mathbf{0}_{(3)}$ and $\mathbf{0}_{(4)}$ denotes zero matrices, which have $(N+T-1)N_r$ rows, and the number of columns of the matrices are determined in the same manner as the per-tone equalizer for the SISO SC-CP system.

In the SIMO SC-CP system, the sliding DFT is performed at each reception antenna, therefore, the corresponding matrix, which stands for all the sliding DFT operation in the receiver, is defined as

$$\bar{\mathbf{F}}_i = \begin{bmatrix} \mathbf{F}_i & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_i & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{F}_i \end{bmatrix}, \tag{16}$$

where $\mathbf{F}_i$ is the same as the matrix defined in (7). Also, the per-tone equalizer weight vector for the ith tone $\mathbf{w}_{i,simo}$ has the size of $N_rT \times 1$.

The per-tone equalizer output vector $\hat{\mathbf{x}}^{(k)}_{simo}$ is given by

$$\hat{\mathbf{x}}^{(k)}_{simo} = \mathbf{F}^H \begin{bmatrix} \mathbf{w}^H_{0,simo}\bar{\mathbf{F}}_0 \\ \vdots \\ \mathbf{w}^H_{N-1,simo}\bar{\mathbf{F}}_{N-1} \end{bmatrix} \mathbf{y}^{(k)}_{simo}. \tag{17}$$

Note that the SIMO per-tone equalizer output has the same expression as the SISO SC-CP system.

By solving a minimization problem of

$$\mathbf{w}^{opt}_{i,simo} = \arg\min_{\mathbf{w}_{i,simo}} E\left[tr\left\{\left(\hat{\mathbf{x}}^{(k)}_{simo} - \mathbf{x}^{(k)}\right)\left(\hat{\mathbf{x}}^{(k)}_{simo} - \mathbf{x}^{(k)}\right)^H\right\}\right], \tag{18}$$

the optimum weight of the ith tone $\mathbf{w}^{opt}_{i,simo}$ in the MMSE sense is obtained as

$$\mathbf{w}^{opt}_{i,simo} = \left[\bar{\mathbf{F}}_i\mathbf{H}_{simo}\mathbf{H}^H_{simo}\bar{\mathbf{F}}^H_i + \sigma_n^2\bar{\mathbf{F}}_i\bar{\mathbf{F}}^H_i\right]^{-1} \cdot \bar{\mathbf{F}}_i\bar{\mathbf{H}}_{simo}\mathbf{f}^H_i, \tag{19}$$

where $\bar{\mathbf{H}}_{simo} = \mathbf{H}_{simo}\left[\mathbf{0}_{N\times N} \;\; \mathbf{I}_N \;\; \mathbf{0}_{N\times N}\right]^T$.

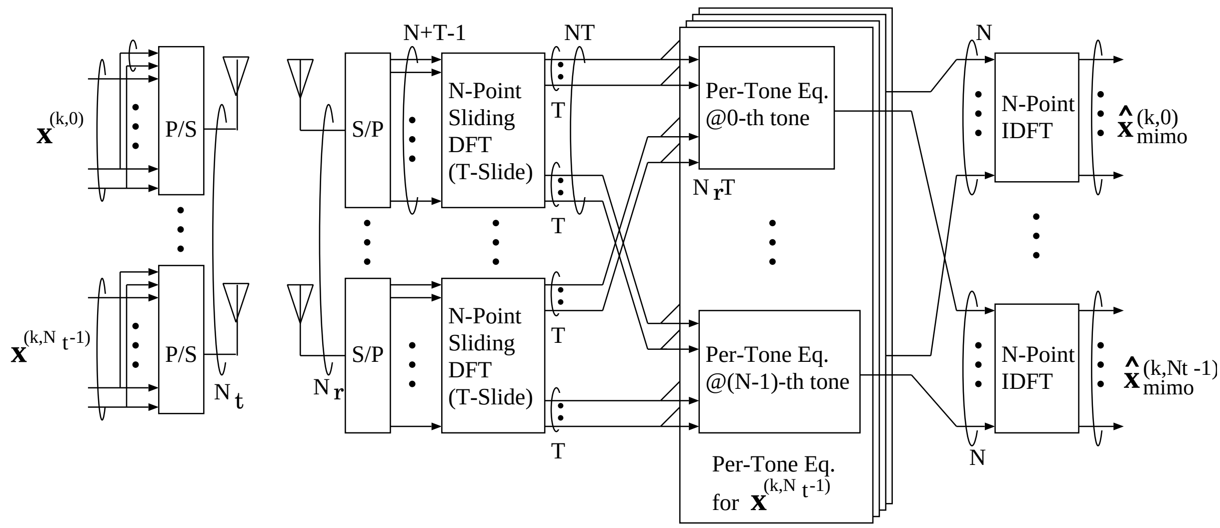

Fig. 6 Configuration of Proposed MIMO SC-CP System

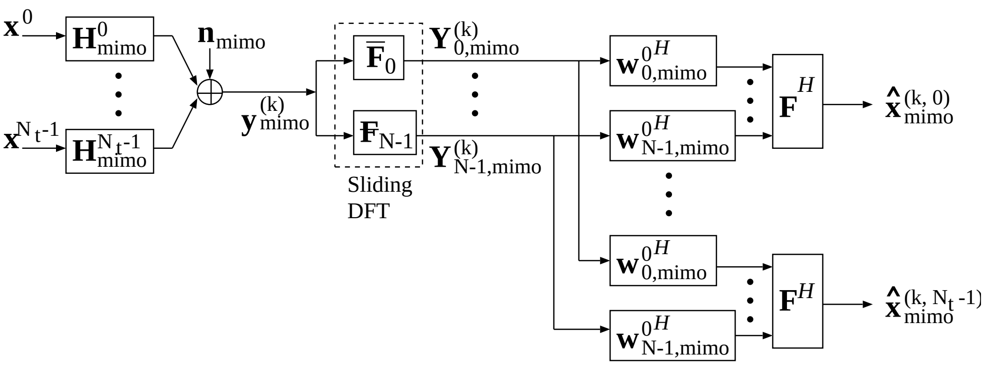

Fig. 7 Schematic Diagram of Proposed MIMO SC-CP System

2.3 MIMO (Multiple-Input Multiple-Output) SC-CP System

Here, we consider a MIMO SC-CP system with N_t transmit antennas and N_r reception antennas. Figs.6 and 7 show a configuration of the MIMO SC-CP system with the proposed per-tone equalizer and the schematic diagram of the MIMO system, respectively.

N_t different signal blocks $\mathbf{x}^{(k,0)}, \ldots, \mathbf{x}^{(k,N_t-1)}$, where $\mathbf{x}^{(k,l)} = [x_0^{(k,l)}, \ldots, x_{N-1}^{(k,l)}]^T$, are simultaneously transmitted from the transmission antennas, therefore, the MIMO receiver has to not only equalize the received signal distorted by the multipath channel but also cancel the co-channel interference. Note that the transmission rate of the MIMO system is N_t times that of the SISO or the SIMO systems.

Let $\mathbf{x}^l$ denote a transmitted signal vector from the lth transmission antenna including the three consecutive signal blocks as $\mathbf{x}^l = \left[\mathbf{x}^{(k-1,l)T}, \mathbf{x}^{(k,l)T}, \mathbf{x}^{(k+1,l)T}\right]^T$. Denoting the received signal at the jth reception antenna $\mathbf{y}^{(k)}_{j,mimo} = [y^{(k),0}_{j,mimo}, \ldots, y^{(k),N+T-2}_{j,mimo}]^T$, we define the received signal vector of the MIMO system as $\mathbf{y}^{(k)}_{mimo} = [\mathbf{y}^{(k)T}_{0,mimo}, \ldots, \mathbf{y}^{(k)T}_{N_r-1,mimo}]^T$. The received signal vector $\mathbf{y}^{(k)}_{mimo}$ can be written as

$$\mathbf{y}^{(k)}_{mimo} = \sum_{l=0}^{N_t-1} \mathbf{H}^l_{mimo}\mathbf{x}^l + \mathbf{n}_{mimo}, \tag{20}$$

where $\mathbf{n}_{mimo}$ is a channel noise vector of size $(N+T-1)N_r \times 1$, and $\mathbf{H}^l_{mimo}$ is a channel matrix between the lth transmission antenna and the receiver defined as

$$\mathbf{H}^l_{mimo} = \left[\begin{array}{ccccccc} & h_L^{0,l} & \ldots & h_0^{0,l} & & \mathbf{0} & \\ & & \ddots & & \ddots & & \\ & \mathbf{0} & & h_L^{0,l} & \ldots & h_0^{0,l} & \\ \mathbf{0}_{(5)} & & \vdots & & \vdots & & \mathbf{0}_{(6)} \\ & h_L^{N_r-1,l} & \ldots & h_0^{N_r-1,l} & & \mathbf{0} & \\ & & \ddots & & \ddots & & \\ & \mathbf{0} & & h_L^{N_r-1,l} & \ldots & h_0^{N_r-1,l} & \end{array}\right]\mathbf{T}. \tag{21}$$

$\{h_0^{j,l}, \ldots, h_L^{j,l}\}$ denotes a channel impulse response between the lth transmission antenna and the jth reception antenna. $\mathbf{0}_{(5)}$ and $\mathbf{0}_{(6)}$ denotes zero matrices, which have $(N + T - 1)N_r$ rows, and the number of columns of the matrices are determined in the same manner as the SISO or SIMO SC-CP systems.

In order to recover the N_t transmitted signal blocks, the MIMO per-tone equalizer has N_t outputs for the received signal vector $\mathbf{y}_{mimo}^{(k)}$. The equalizer output, which corresponds to the lth transmission antenna, $\hat{\mathbf{x}}_{mimo}^{(k,l)}$ is given by

$$\hat{\mathbf{x}}_{mimo}^{(k,l)} = \mathbf{F}^H \begin{bmatrix} \mathbf{w}_{0,mimo}^{lH} \bar{\mathbf{F}}_0 \\ \vdots \\ \mathbf{w}_{N-1,mimo}^{lH} \bar{\mathbf{F}}_{N-1} \end{bmatrix} \mathbf{y}_{mimo}^{(k)}, \tag{22}$$

where $\mathbf{w}_{i,mimo}^l$ $(N_r T \times 1)$ is an equalizer weight vector of the ith tone for the transmitted signal from the lth transmission antenna.

By solving the minimization problem of

$$\mathbf{w}_{i,mimo}^{l,opt} = \arg \min_{\mathbf{w}_{i,mimo}^l} E\left[tr\left\{ \left(\hat{\mathbf{x}}_{mimo}^{(k,l)} - \mathbf{x}^{(k,l)}\right) \left(\hat{\mathbf{x}}_{mimo}^{(k,l)} - \mathbf{x}^{(k,l)}\right)^H \right\} \right], \tag{23}$$

the optimum weight of the ith tone for the transmitted signal from the lth transmission antenna $\mathbf{w}_{i,mimo}^{l,opt}$ in the MMSE sense is obtained as

$$\mathbf{w}_{i,mimo}^{l,opt} = \left[\bar{\mathbf{F}}_i \left(\sum_{l=0}^{N_t-1} \mathbf{H}_{mimo}^l \mathbf{H}_{mimo}^{lH} \right) \bar{\mathbf{F}}_i^H + \sigma_n^2 \bar{\mathbf{F}}_i \bar{\mathbf{F}}_i^H \right]^{-1} \cdot \bar{\mathbf{F}}_i \bar{\mathbf{H}}_{mimo}^l \mathbf{f}_i^H, \tag{24}$$

where $\bar{\mathbf{H}}_{mimo}^l = \mathbf{H}_{mimo}^l \left[\mathbf{0}_{N\times N} \;\; \mathbf{I}_N \;\; \mathbf{0}_{N\times N}\right]^T$.

3. Computer Simulation

In order to evaluate the BER performance of the proposed per-tone equalization methods, computer simulations have been conducted. The equalizer weights of the SISO, SIMO, and MIMO SC-CP systems are calculated using (13), (19), and (24), respectively. System parameters used in the simulations are listed in Table 1. QPSK scheme with coherent detection is employed for the modulation/demodulation scheme. The information block size N and the length of the guard interval K are set to be 64 and 16, respectively. In the simulations, the number of taps at each tone T is changed from 1 to 5. Moreover, we have employed a 9-path frequency selective Rayleigh fading channel with the channel order of $L = 20$. This means that the guard interval K is insufficient by 4 symbols length. To be more precise, we have obtained the BER performances by averaging the BERs evaluated by using 10,000 realizations of channel impulse response. The channel responses are generated as follows. Fixing the number of the paths to be 9, the temporal position of each path is randomly determined by following uniform distribution of [0:20] symbols delay. The amplitude and the phase of each path are also randomly determined by following Rayleigh distribution and uniform distribution of [0:360) deg respectively. Therefore, since $K = 16$, the average power of the delayed signals beyond the guard interval is 20 % of all the received signal power. Furthermore, the channel response is assumed to be known to the receiver.

Table 1 System Parameters

Mod/Demod Scheme	QPSK
Symbols/block N	64
Guard Interval K	16
# of taps/tone T	1-5
Channel Order L	20
Channel model	9-path Rayleigh Fading Channel
Channel estimation	Perfect

Fig.8 shows the BER performance versus the ratio of the energy per bit to the noise power density (E_b/N_0) of the SISO SC-CP system with the proposed per-tone equalizer for the different number of taps per tone($T = 1 - 5$). Note that the per-tone equalizer with $T = 1$ corresponds to the conventional one-tap FDE. From the figure, we can see that the proposed per-tone equalizer with $T \geq 2$ can significantly improve the BER performance. The BER performance of the proposed equalizer is improved as the number of taps per-tone T increases, however, the improvement is saturated at $T = 4$. This is because the channel model used in the simulation includes up to 4 symbols delay beyond the GI and the proposed equalizer with the number of taps more than or equal to 4 can take into consideration all the source of the interference due to the insufficient GI. Also, this means that the number of taps of the per-tone equalizer could be determined to be $L - K$. The power of the delayed signal beyond the guard interval critically affects the performance gain of the proposed system from the performance of the conventional equalizer. This is because the BER performance of the proposed equalizer coincides with that of the conventional MMSE based one-tap FDE if the length of the guard interval is sufficient and the performance gain of the proposed methods in the case of insufficient guard interval largely depends on the performance degradation of the conventional equalizer due to the delayed signals beyond the guard interval. Therefore, if the power of the delayed signals beyond guard interval is large, we can expect large performance gain by the proposed equalizer, and vice versa.

Fig.9 shows the BER performance versus the E_b/N_0 per reception antenna element of the SIMO SC-CP system with the proposed per-tone equalizer for the

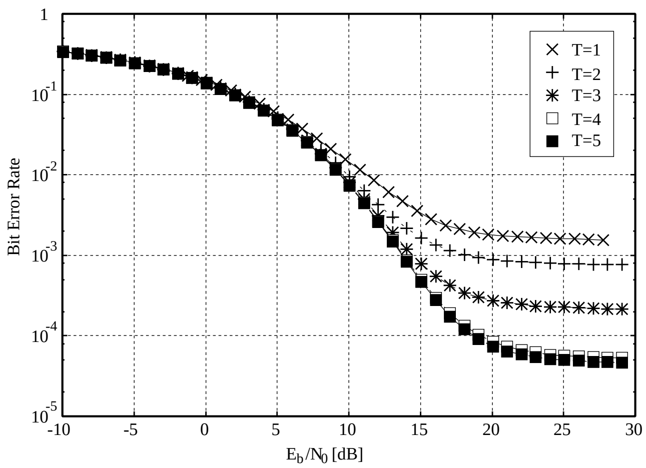

Fig. 8 BER Performance: SISO SC-CP System

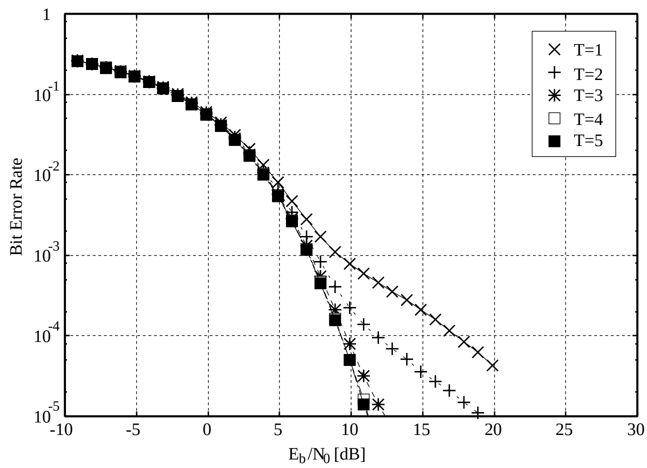

Fig. 9 BER Performance: SIMO SC-CP System

different number of taps per tone($T = 1 - 5$). The number of the reception antenna $N_r$is set to be 2. In the Figure, the SIMO per-tone equalizer with $T = 1$ corresponds to a conventional SC-CP system with a post-DFT type adaptive antenna array. By comparing the BER performance of the conventional adaptive antenna array system as that of the SISO per-tone equalizer with $T = 1$ in Fig. 8, we can see that the adaptive antenna array can improve the performance. This is because the adaptive antenna array can remove the delayed signals beyond the guard interval by forming nulls toward the delayed signals. However, the proposed per-tone equalizer with $T \geq 2$ can further improve the BER performance by utilizing the power of the delayed signals not only within the guard interval but also beyond the guard interval.

Fig.10 shows the BER performance versus the E_b/N_0 per reception antenna element of the MIMO SC-CP system with the proposed per-tone equalizer for the difference number of taps per tone($T = 1 - 5$). The number of the transmission antenna N_t and the reception antenna N_r are set to be 2 and 2 respectively, therefore, the transmission rate of the MIMO system is twice as that of the SISO or SIMO systems. When $T = 1$, the proposed system is the same as the MIMO system proposed in [12]. We can see that the proposed scheme with $T \geq 2$ outperforms the conventional MIMO SC-CP system.

From all the results, it can be concluded that the proposed per-tone equalization schemes can significantly improve the BER performance of the SISO, SIMO, and MIMO SC-CP systems with insufficient guard interval and the number of taps per tone T can be selected to be $L - K$.

4. Conclusion

We have proposed per-tone equalization methods for the SC-CP systems with the insufficient guard interval.

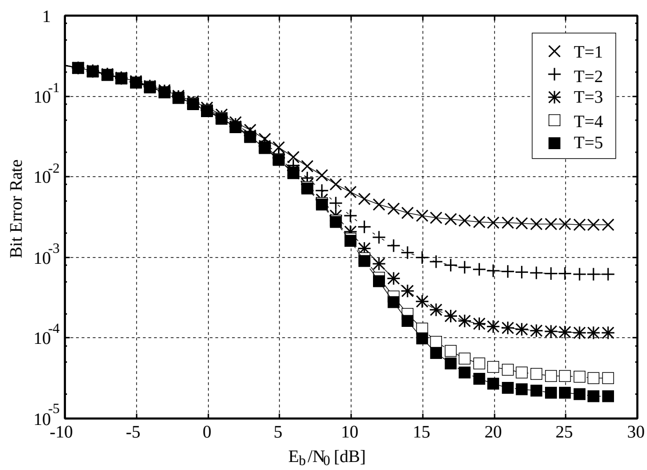

Fig. 10 BER Performance: MIMO SC-CP System

MMSE based optimum weights of the per-tone equalizers are derived for the SISO, SIMO, and MIMO SC-CP systems. With the data modeling methods used in this paper, unified treatment of the per-tone equalizers for the SC-CP systems is possible. Moreover, computer simulations are conducted in order to evaluate the BER performance of the proposed systems with the insufficient guard interval. From all the results, it can be concluded that the proposed scheme can significantly improve the BER performance of the SISO, SIMO, and MIMO SC-CP systems with the insufficient guard interval.

In this paper, we have assumed that the channel state information is known to the receiver. In practical situations, however, channel and noise variance estimation is required to determine the equalizer weights. Although we have already proposed a channel estimation method for the SC-CP system with insufficient guard interval[13], the optimum design of pilot signal for the channel estimation or the noise variance estimation method will be our future work.

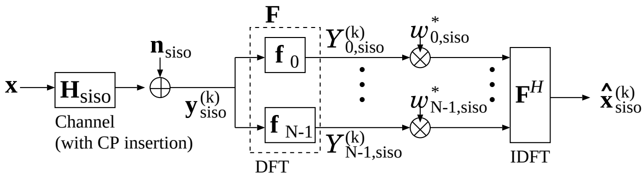

Fig. A· 1 Schematic Diagram of Conventional One-Tap FDE

References

[1] S. Hara and P. Prasad, "Overview of multicarrier CDMA," *IEEE Commun. Mag.*, vol.35, pp.126-133, Dec. 1997.

[2] H. Sari, G. Karam and I. Jeanclaude, "Transmission Techniques for Digital Terrestrial TV Broadcasting," *IEEE Commun. Mag.*, vol.33, pp.100-109, Feb. 1995.

[3] J. A. C. Bingham, "Multicarrier Modulation for Data Transmission: An idea whose Time Has Come," *IEEE Commun. Mag.*, vol.28, pp.5-14, May 1990.

[4] D. Falconer et al., "Frequency Domain Equalization for Single-Carrier Broadband Wireless System," *IEEE Commun. Mag.*, vol.40, pp.58-66, Apr. 2002.

[5] Z. Wang and G. B. Giannakis, "Wireless Multicarrier Communications," *IEEE Signal Processing Mag.*, vol.17, pp.29-48, May 2000.

[6] P. J. W. Melsa, R. C. Younce, and C. E. Rohrs, "Impulse Response Shortening for Discrete Multitone Transceivers," *IEEE Trans. Commun.,* vol. 44, pp. 1662-1672, Dec. 1996.

[7] B. Farhang-Boroujeny and M. Ding, "Design Methods for Time-Domain Equalizers in DMT Transceivers," *IEEE Trans. Commun.,* vol. 49, pp. 554-562, Mar. 2001.

[8] G. Arslan, B. L. Evans, and S. Kiaei, "Equalization for Discrete Multitone Receivers To Maximize Bit Rate," *IEEE Trans. Signal Processing,* vol. 49, pp. 3123-3135, Dec. 2001.

[9] K. Van Acker, G. Leus, M. Moonen, O. van de Wiel, and T. Pollet, "Per Tone Equalization for DMT-Based Systems," *IEEE Trans. Commun.*, vol. 49, pp. 109-119, Jan. 2001.

[10] G. Leus and M. Moonen, "Per-Tone Equalization for MIMO OFDM Systems," *IEEE Trans. Signal Processing*, vol. 51, pp. 2965-2975, Nov. 2003.

[11] E. Jacobsen and R. Lyons, "The Sliding DFT," *IEEE Signal Processing Mag.*, vol. 20, no. 2, pp. 74-80, Mar. 2003.

[12] P. Vandenameele, L. Van der Perre, B. Gyselinckx, M. Engels, M. Moonen, and H. De Man, "A Single-carrier Frequency-domain SDMA Basestation," *Proc. of ICASSP 2000*, vol. 6, pp. 3714-3717, June 2000.

[13] K. Hayashi and H. Sakai, "A Simple Interference Elimination Scheme for Single Carrier Block Transmission with Insufficient Cyclic Prefix," *Proc. of WPMC 2004*, vol. 2, pp. 577-581, Sept. 2004.

Appendix A: Proposed Equalizer with $T = 1$ and Sufficient Cyclic Prefix

Here, we show that in the case of $T = 1$ and $L \leq K$, the proposed weight of the SISO equalizer perfectly coincides with the MMSE optimum weight of the one-tap FDE.

When $T = 1$ and $L \leq K$, the matrix $\mathbf{H}_{siso}$ can be written as

$$\mathbf{H}_{siso} = \begin{bmatrix}\mathbf{0}_{N\times N} & \mathbf{C}_{siso} & \mathbf{0}_{N\times N}\end{bmatrix}, \quad \text{(A· 1)}$$

where $\mathbf{C}_{siso}$ is an $N \times N$ circulant matrix defined as

$$\mathbf{C}_{siso} = \begin{bmatrix} h_0 & 0 & \dots & 0 & h_L & \dots & h_1 \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & h_L \\ h_L & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \dots & 0 & h_L & \dots & \dots & h_0 \end{bmatrix}. \quad \text{(A· 2)}$$

Also, when $T = 1$ and $L \leq K$, $\mathbf{F}_i$ is equal to $\mathbf{f}_i$ and $\bar{\mathbf{H}}_{siso}$ becomes $\mathbf{C}_{siso}$ defined above. Therefore, the ith weight of the proposed SISO equalizer $w_{i,siso}^{opt}$ can be simplified as

$$\begin{aligned} w_{i,siso}^{opt} &= \left[\mathbf{F}_i\mathbf{H}_{siso}\mathbf{H}_{siso}^H\mathbf{F}_i^H + \sigma_n^2\mathbf{F}_i\mathbf{F}_i^H\right]^{-1}\mathbf{F}_i\bar{\mathbf{H}}_{siso}\mathbf{f}_i^H, \\ &= \frac{\mathbf{f}_i\mathbf{C}_{siso}\mathbf{f}_i^H}{\mathbf{f}_i\mathbf{C}_{siso}\mathbf{C}_{siso}^H\mathbf{f}_i^H + \sigma_n^2}. \end{aligned} \quad \text{(A· 3)}$$

Note that the proposed weight $w_{i,siso}^{opt}$ is scalar in this case.

It is well known that any circulant matrix can be diagonalized by DFT matrix[5] as $\mathbf{C}_{siso} = \mathbf{F}^H\mathbf{\Lambda}\mathbf{F}$, where $\mathbf{\Lambda}$ is a diagonal matrix, whose diagonal elements $\{\lambda_0, \dots, \lambda_{M-1}\}$ are defined by the DFT of $[h_0, \dots, h_L, \mathbf{0}_{(M-L-1)\times 1}]^T$. By substituting $\mathbf{C}_{siso} = \mathbf{F}^H\mathbf{\Lambda}\mathbf{F}$, $\mathbf{w}_{i,siso}^{opt}$ can be further calculated as

$$\begin{aligned} w_{i,siso}^{opt} &= \frac{\mathbf{f}_i\mathbf{F}^H\mathbf{\Lambda}\mathbf{F}\mathbf{f}_i^H}{\mathbf{f}_i\mathbf{F}^H\mathbf{\Lambda}\mathbf{F}\mathbf{F}^H\mathbf{\Lambda}^H\mathbf{F}\mathbf{f}_i^H + \sigma_n^2}, \\ &= \frac{\lambda_i}{|\lambda_i|^2 + \sigma_n^2}. \end{aligned} \quad \text{(A· 4)}$$

As shown in Fig.2, the received signal is multiplied by the Hermitian transpose of $\mathbf{w}_{i,siso}^{opt}$ in the proposed system, therefore, Eq. (A· 4) is exactly the same as the weight of the conventional MMSE based one-tap FDE[5].

Fig. A· 1 shows the configuration of the conventional MMSE based one-tap equalizer. The major difference in the configuration between the two equalizers is that, in the conventional equalizer, the received signal is fed into not the sliding DFT but the DFT. Therefore, the frequency domain received signal per-tone becomes a scalar and hence the number of weight per-tone is one.

Kazunori Hayashi received the B.E., M.E. and Ph.D. degrees in communication engineering from Osaka University, Osaka, Japan, in 1997, 1999 and 2002, respectively. Since 2002, he has been with the Department of Systems Science, Graduate School of Informatics, Kyoto University. He is currently an Assistant Professor there. His research interests include digital signal processing for communications systems.

Hideaki Sakai received the B.E. and D.E. degrees in applied mathematics and physics from Kyoto University, Kyoto, Japan, in 1972 and 1981, respectively. From 1975 to 1978, he was with Tokushima University. He is currently a Professor in the Department of Systems Science, Graduate School of Informatics, Kyoto University. He spent 6 months from 1987 to 1988 at Stanford University as a Visiting Scholar. His research interests are in the areas of adaptive and statistical signal processing. He served as an associate editor of IEEE Trans. Signal Processing from Jan. 1999 to Jan. 2001.