A Power Allocation Scheme for MIMO OFDM Systems

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***Abstract*— The growing demand of multimedia services and the growth of broadband Internet related contents lead to increasing interest to high speed communications. This is because the higher bit rate and higher quality applications must be provided via the Internet such as huge data downloading, high quality streaming videos. On the other hand, the requirement for wide bandwidth and flexibility imposes the use of efficient transmission methods that would fit well to the characteristics of wideband channels especially in wireless environment where the channel is very hostile. Combining Multi-Input Multi-output (MIMO) and Orthogonal Frequency Division Multiplexing (OFDM) with Guard Interval (GI) is a highly promising method to achieving required data rate for real time application. However, in a frequency selective fading environment, equally sharing the power transmission in the space domain does not optimize the global transmission. In this publication, we propose to adapt the power allocation in the space domain (i.e. through the different transmit antennas) in function of the channel variations and optimization process based on the optimality of the global Bit Error Rate (BER). Simulation results show significant gains for several antenna configurations.**

***Keywords*- OFDM, MIMO, Lagrangian method, global BER optimization.**

I. INTRODUCTION

In wireless environment the signal is propagating from a transmitter to a receiver along with a number of different paths, collectively referred as multipath. While propagating the signal power drops due to three effects: path loss, macroscopic fading and microscopic fading. Fading of the signal can be mitigated by different diversity techniques. In recent years, interest in the realization of high data rate wireless communication systems, such as wireless local area networks (LANs) has been increasing. Among IEEE 802.11 wireless LAN standard, 802.11a [1] and 802.11g [2] systems employ OFDM, which offer high spectral efficiency and superior tolerance to multi-path fading. Another trend is the interest in the field of multi-antenna processing technique. In rich multipath environment, space division multiplexing (SDM) with MIMO systems can increase the transmission rate [3]-[4] and has enormous communication capacity because of its spectral efficiency [5]. In a spatially uncorrelated frequency selective fading channel environment, spatial domain channel variation can significantly modify the performance of the MIMO-OFDM transmission. In this paper, we propose a method to efficiently allocate transmit power in function of the channel conditions and the global optimization of the BER. In Section II, we describe the MIMO OFDM systems and the associate signal model. In Section III, we introduce and describe the proposed power allocation scheme. Section IV gives the simulation results obtained through frequency selective fading channels over QPSK modulation and conclusions are drawn in Section V.

II. SYSTEM DESCRIPTION

A. MIMO OFDM signal

The principle of OFDM transmission scheme [6] is to reduce bit rate of each sub-carrier and also to provide high bit rate transmission by using a number of those low bit rate sub-carriers. Frequency bandwidth is divided into small ranges and each of them is handled by these low rate sub-carriers. Here, it is important that the sub-carriers are orthogonal to each other. To obtain this property, the sub-carrier frequencies must be spaced by a multiple of the inverse of symbol duration. Multi-carrier modulation system can provide immunity against frequency selective fading because each carrier goes through non-frequency selective fading. However, the channel must be estimated and corrected for each sub-carrier. Given the system description of the OFDM system, we can develop a MIMO-OFDM signal model. In this paper, we will need both time-domain and frequency-domain model. Suppose that a communication system consists of N_t TX and N_r RX antennas, denoted as $N_t \times N_r$ system, where the transmitter at a discrete time interval t sends a N_t-dimensional complex vector and the receiver receives an N_r-dimensional complex vector. An OFDM system transmits N modulated data symbols in the i-th OFDM symbol period through N sub-channels. The transmitted baseband OFDM signal for the i-th block symbol, is expressed as [7]:

$$s_{i,n}^{(p)} = \frac{1}{\sqrt{N}} . \sum_{k=0}^{N-1} x_{i,k}^{(p)} . \exp\left\{ j \frac{2\pi . nk}{N} \right\} \tag{1}$$

where $x_{i,k}^{(p)}$ is the modulated data symbol of the p-th transmit antenna for the i-th OFDM symbol. To combat ISI and Inter Carrier Interference (ICI), Guard Interval (GI) [8] such as

Cyclic Prefix (CP) or Zero Padding (ZP) is added to the OFDM symbols. In the case of CP, the last N_g samples of every OFDM symbol are copied and added to the heading part. The transmit signal can be described as follow:

$$\tilde{s}_{i,n}^{(p)} = \begin{cases} s_{i,N-Ng+n}^{(p)}, & \text{for } 0 \leq n < N_g \\ s_{i,n-Ng}^{(p)}, & \text{for } N_g \leq n < N + N_g \end{cases} \tag{2}$$

We assume that the system is operating in a frequency selective Rayleigh fading environment [9] and the communication channel remains constant during a packet transmission. Data frame duration is assumed to transmit within the coherent time of the wireless system. In this case, channel variations remain constant during on frame transmissions and may change between consecutive frame transmissions. We suppose that the fading channel can be modeled by a discrete-time baseband equivalent $(L-1)$-th order finite impulse response (FIR) filter where L represents time samples corresponding to the maximum delay spread. In addition, an Additive White Gaussian Noise (AWGN) with N_r independent and identically distributed (iid) zero mean, complex Gaussian elements is assumed. When the maximum delay spread does not exceed GI, since ISI does not occur on MIMO OFDM symbol basis, the frequency domain MIMO OFDM signal after removal of GI is described by:

$$y_{j,m}^{(q)} = \sum_{p=0}^{N_t-1} h_m^{(q,p)}.x_{j,m}^{(p)} + n_{j,m}^{(q)} \tag{3}$$

where $y_{j,m}^{(q)}$ is the received signal at the q-th received antenna for the j-th OFDM symbol and the m-th sub-carrier and $h_{j,m}^{(q,p)}$ is the channel parameter from the p-th transmitting antenna to the q-th receiving antenna which composes the MIMO channel matrix. In addition, $n_{j,m}^{(q)}$ denotes the AWGN for the q-th received antenna. Thus it results in a frequency-flat-fading signal model per sub-carrier. For simplicity, without losing any generality, we will omit writing the index for both the sub-carrier and the symbol indicators. Hereafter, the received signal can simply be written as:

$$\mathbf{Y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{4}$$

where **n** represents zero mean, complex AWGN with covariance matrix equals to:

$$\mathbf{E}[\mathbf{n}\mathbf{n}^H] = \sigma_n^2 \mathbf{I}_{N_r} \tag{5}$$

B. MIMO OFDM detection scheme

Let now recall the linear MIMO detection with respect to the zero forcing (ZF) and to the minimum mean square error (MMSE) criteria [10]. In this section, we denote $\mathbf{G} = \{\mathbf{G}_m^{(l,n)}\}_{\forall l,n,m}$ the matrix representation of the detection scheme.

1) Zero Forcing Detector (ZF): In a ZF linear detector, the received signal vector is multiplied with a filter matrix which is a pseudo inverse of the channel response.

$$\mathbf{G} = (\mathbf{H}^H\mathbf{H})^{-1}.\mathbf{H}^H \tag{6}$$

2) Minimum Mean Square Detector (MMSE): The MMSE detector minimizes the mean square error between the actually transmitted symbols and the output of the linear detector which is defined by:

$$\mathbf{G} = (\alpha.\mathbf{I}_{N_r} + \mathbf{H}^H\mathbf{H})^{-1}.\mathbf{H}^H \tag{7}$$

where α is equal to $\frac{1}{SNR}$.

III. POWER ALLOCATION SCHEME

A. Proposed principle

The basic principle of the space domain power allocation for MIMO OFDM signal is to perform spatial domain optimization of the transmit power in function of the channel state information (CSI) and the expression of the global BER for the specific QPSK modulation. By simply performing transmit power allocation from the estimate value of the BER, we propose to optimize the transmit power allocation by performing Lagrangian method on the BER expression. Constraint is added in order to keep the global transmit power at the transmitting part constant.

B. Mathematical description

By definition, the BER is a function of the SNR (i.e. transmit power denoted $p_{l,m}$ for the l-th transmit antenna and the m-th subcarrier, and channel condition). In [11], a general formula describing this relation in a flat fading channel environment is proposed. Typically, the BER for QPSK modulation can be simply estimated as follows:

$$f(\beta_{l,m}, p_{l,m}) \approx a \times \exp\left\{ -b \times \beta_{l,m} \times p_{l,m} \right\} \tag{8}$$

where the different elements are equal to:

$$\beta_{l,m} = \frac{1}{(2^{N_m}-1).\sigma_n^2.\sum_{n=0}^{N_r-1} |\mathbf{G}_m^{(l,n)}|^2} \tag{9}$$

N_m represents the number of bit per modulated symbol (i.e. $N_m = 2$ for QPSK modulation).

Proposed power allocation consists of optimizing the BER function described below by estimating optimum power allocation [12]. Furthermore, to obtain realistic solution, we add the constraint which can be described as follows:

$$\sum_{l=0}^{N_t-1} p_{l,m} = N_t.\overline{P}_m \tag{10}$$

where $\overline{P}_m$ denotes the average transmit power on the subcarrier m.

The average BER becomes minimal when the BER is minimized for each given channel state and one possibility to solve this optimization problem is to apply the Lagrangian procedure described as:

$$\begin{aligned} J(p_{0,m},....,p_{N_t-1,m}) &= \frac{1}{N_t}\sum_{l=0}^{N_t-1} f(\beta_{l,m},p_{l,m}) \\ &+ \lambda \times (\sum_{l=0}^{N_t-1} p_{l,m} - N_t \times \overline{P}_m) \end{aligned} \quad (11)$$

Then, optimal solutions are obtained by solving for each transmit antenna:

$$\begin{cases} \frac{1}{N_t}.\frac{\partial}{\partial p_{l,m}}\left(\sum_{l=0}^{N_t-1} f(\beta_{l,m},p_{l,m})\right) + \lambda = 0 \\ \sum_{l=0}^{N_t-1} p_{l,m} - N_t.\overline{P}_m = 0 \end{cases} \quad (12)$$

By introducing the explicit estimation of the BER in the set of equations, we can write for each transmit antenna that:

$$\begin{cases} \frac{1}{N_t} \times (-a.b.\beta_{l,m}) \times \exp(-b.\beta_{l,m}.p_{l,m}) + \lambda = 0 \\ \sum_{l=0}^{N_t-1} p_{l,m} - N_t.\overline{P}_m = 0 \end{cases} \quad (13)$$

with a=0.2 and b=1.5.

After calculation and rearrangement, we finally obtain the following general solution:

$$\begin{aligned} p_{l,m} = &\left[1 + \sum_{\substack{u=0 \\ u\neq l}}^{N_t-1} \frac{\beta_{l,m}}{\beta_{u,m}}\right]^{-1} \\ &\times \left[N_t.\overline{P}_m + \frac{1}{b} \times \sum_{\substack{u=0 \\ u\neq l}}^{N_t-1} \frac{1}{\beta_{u,m}} \times log\left(\frac{\beta_{l,m}}{\beta_{u,m}}\right)\right] \end{aligned} \quad (14)$$

Due to the simplicity of this Lagrangian calculation, a formal solution of Eq. 14 is obtained. However, we need to add a constraint when the output of the Lagrangian optimization does not reflect any physical solution, typically when we obtain: $p_{l,m} \leq 0$. In this case we propose to apply the conventional scheme.

IV. EXPERIMENTATION

We now evaluate the performance of the proposed power allocation for MIMO-OFDM scheme in a mutli-path fading environment. Main simulation parameters are presented in the Table I. We assume perfect knowledge of the channel variations both at the transmitting and receiving parts. Effect of the number of transmit antennas is highlighted in the different simulation results presented in this section.

Fig.1, 2, and 3 show the Bit Error Rate (BER) versus the average total received SNR (dB) for different antenna configurations. Both conventional and proposed schemes are plotted in these figures and results are presented for both ZF and MMSE detection schemes.

TABLE I

SIMULATION PARAMETERS

Carrier Frequency	2.4 GHz
Bandwidth	20 MHz
Modulations	QPSK
Channel encoder	No coding
Channel estimation	Perfect CSI
Number of data subcarrier	64
Guard Interval length	16
Channel model	5-path, Rayleigh Fading
Sample period	0.05μs
Number of data packet	50
(Nt,Nr) configuration	(4,4), (8,8) and (12,12)

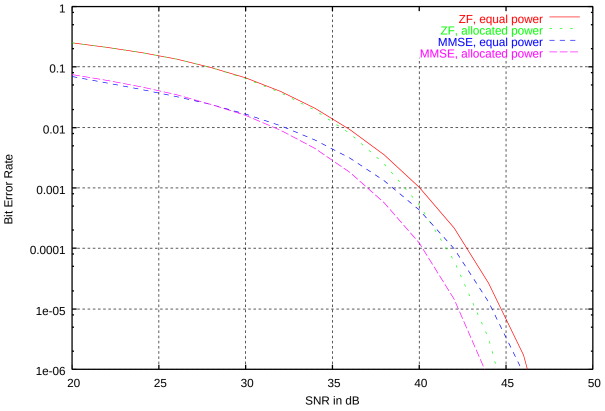

Fig. 1. Performance Simulation for Tx=Rx=4

In the Fig. 1, the benefit of performing the proposed scheme is highlighted. At average BER=10^{-5}, in the $N_t = N_r = 4$ configuration, 1.6dB gain is obtained in the case of ZF detection scheme. Gain becomes more significant for MMSE detection scheme. For instance, at BER=10^{-5}, 2.1dB gain is obtained. For low average total received SNR, at BER=10^{-2} proposed and conventional schemes have similar performance. This property comes from the condition that only realistic solutions are kept into consideration and for relatively low total SNR, proposed scheme is not used (i.e. no solution is available to allocate specific power on each transmit antenna). Gain of the proposed method is highlighted from average BER=10^{-3}.

In the Fig. 2, in the case $N_t = N_r = 8$, gain obtained by performing proposed power allocation scheme becomes even larger compared to the case $N_t = N_r = 4$. At average BER=10^{-5}, 2.8dB gain is obtained for ZF detection case and gain becomes equal to 3.2dB for MMSE detection scheme.

In the Fig. 3, in the case $N_t = N_r = 12$, gain obtained by performing proposed power allocation scheme becomes relatively important. At average BER=10^{-5}, 3.1dB gain is obtained for ZF detection case and gain becomes equal to 3.6dB for MMSE detection scheme. In addition, due to the antenna configuration, for high total SNR, compare to the ZF scheme, the effect of the MMSE detection is very limited in the equal power transmission scheme. However, in the power

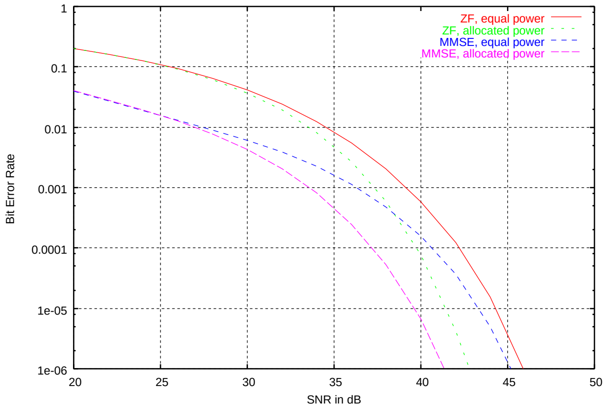

Fig. 2. Performance Simulation for Tx=Rx=8

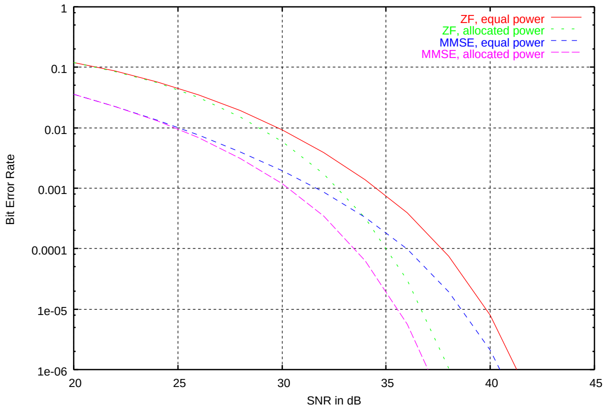

Fig. 3. Performance Simulation for Tx=Rx=12

allocation case, we can see that gain obtained by performing MMSE detection (compared with the ZF scheme) is still significant, about 0.5dB gain at BER=10^{-5}.

TABLE II

REQUIRED TOTAL SNR FOR SEVERAL CHANNEL CONFIGURATIONS IN THE CASE $N_t = N_r = 4$

Number of path	equal power	allocated power
1	33.1	31.5
5	44.8	43.2
10	42.1	40.5
15	41.1	39.5

Table II shows the required total SNR in dB for the ZF detection scheme in the specific antenna configuration $N_t = N_r = 4$ at $BER = 10^{-5}$. The main fact is that the channel multipath configuration does affect the global performance of the equal power and allocated power schemes. However, the channel multipath condition does not affect the gain obtained by performing the proposed scheme, which is in average between 1.5dB and 2dB.

Table III presents the required average SNR in dB to reach $BER = 10^{-5}$ in the configuration $N_t = N_r = 8$ and the detection scheme is ZF. These results highlight that for any type of channel conditions, proposed method outperforms the conventional scheme (i.e. equal transmit power in the space domain). In average, 3dB gain is obtained.

TABLE III

REQUIRED TOTAL SNR FOR SEVERAL CHANNEL CONFIGURATIONS IN THE CASE $N_t = N_r = 8$

Number of path	equal power	allocated power
1	31.6	28.5
5	44.5	41.7
10	42.4	39.5
15	41.2	38.3

V. CONCLUSION

In this paper, a novel power allocation processing for MIMO based transmission has been presented. The basic procedure consists of allocating transmit power in function of the channel conditions and the BER requirement. The simulation results have shown promising results in term of BER for several sets of antenna configuration. Future works would include the introduction of the error in the channel estimation and the generalization of this proposed power allocation method for different QAM modulations.

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