On the Systematic Block Size Adaptation for Coded Single Carrier Overlap Frequency Domain Equalization Systems

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Abstract— **In this paper, we propose a systematic design of the overlap frequency domain equalizer (FDE) for SC transmissions. Based on the description of the inter-symbol interference (ISI) and the inter-carrier interference (ICI), we evaluate the signal-to-interference-and-noise-ratio (SINR) at the FDE output and adjust the block size of the overlap FDE. The proposed method consists of a simple procedure to evaluate the utility part of the equalized signal. The systematic approach can be extremely useful to adjust the block size. Simulation results validate the proposed systematic design of the overlap FDE and show that the proposed scheme achieve approximatively the same bit error rate (BER) performances than of the SC FDE with well-designed guard interval insertion for both coded cases with various modulation levels.**

Keywords- **Single carrier, overlap frequency domain equalization, SINR**

I. Introduction

Since the 90's, single carrier transmission with frequency domain equalization (SC-FDE) has been thoroughly studied and has been drawing much attention due to the effectiveness and simplicity of the transceiver [1]. When the SC-FDE is used with cyclic prefix, the SC-FDE not only outperforms the orthogonal frequency division multiplexing (OFDM) system in the absence of channel coding, but also loosens the requirement on the power amplifiers due to the low peak to average power ratio (PAPR) of the transmitted signals. Recently, both the OFDM and the SC-FDE have been seen as complementary solutions to each other since the OFDM and the SC-FDE can coexist in a dual-mode multiple access system [2] allowing some parts of the signal processing to shift from the mobile to the base station (BS) in the uplink transmission. However, guard interval (GI) is considered as a main limitation to achieve highly efficient signal transmission. For example, for IEEE 802.11a/g [14]-[15] based transmission, the GI represents 25% of the bandwidth occupation. Transmission with insufficient GI insertion has been studied by several research groups [3]-[4] and several authors have proposed specific frame structure to reduce the impact of the GI without creating interference [5]-[9]. Several years ago, a general equalization for multi-carrier (MC) transmission without any GI has been proposed in [7]-[6] and, more recently, [8] proposed similar concept. Their solution is to perform overlap frequency domain equalization (overlap FDE). In [9], the authors propose to extend those results to direct sequence code division multiple access (DS-CDMA) transmissions. The basic idea of the SC overlap FDE considers the possibility of eliminating the guard interval assistance, when transmitting sequences of SC blocks. The principle consists of extending the size of the Fourier transform, denoted the window size, at the equalization part in order to spread the interference effects at the header and the tailor part of the processed signal. By cutting the two extreme parts, effect of the interference can be mitigated.

The rest of this article is organized as follows. Section II shows the system description of the SC-FDE. In Section III, we introduce the SINR estimation. Section IV describes our method to efficiently estimate the block size associated the overlap FDE equalization. Section V presents numerical results over QPSK and QAM modulations and channel coded systems. Finally, conclusions are drawn in Section 6.

II. System Description

A. Transmit Signal and Channel Representation

SC system is a traditional digital transmission scheme in which data are transported as a fixed symbol rate serial stream of amplitude and/or phase modulated pulses. At the transmitter, binary input data are first encoded by a convolutional encoder and then transmitted in the air. Transmit sequence can be expressed as

$$s(t) = \sum_{v=0}^{V-1} x_v \cdot V(t - v \cdot T_d), \qquad (1)$$

where the term u(t), x_v, V and T_d are respectively the step function defined within the duration of $[0, T_d]$, the modulated data sequence, the number of modulated symbols by transmit stream and the symbol duration. The signal $s(t)$ is then transmitted through a multipath wireless channel characterized by

$$h(t,\tau) = \sum_{i=0}^{P-1} \alpha_i(t) \cdot c(\tau - \tau_i) \tag{2}$$

where $h(t,\tau)$ is the channel response, P the number of paths, $\alpha_i(t)$, τ_i the complex amplitude and the delay of the i-th path, respectively, and $c(t)$ is a shaping pulse. To simplify the derivations we will assume $c(t) = \delta(t)$, where

$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & else \end{cases} \tag{3}$$

In the remainder of the paper, we will assume that the channel is deterministic and time independent [17], i.e. $\alpha_i(t) = \alpha_i$, which yields:

$$h(t,\tau) \equiv h(\tau) = \sum_{i=0}^{P-1} \alpha_i c(\tau - \tau_i) \tag{4}$$

For a time-independent channel, the received signal $r(t)$ in the time domain is given by

$$r(t) = s(t) * h(t) + e(t) \tag{5}$$

where " $*$ " denotes the convolution and $e(t)$ is an additive white Gaussian noise (AWGN) identically distributed (iid) zero mean, σ_n^2 variance [17].

Without any lost of generality, we assume, in this paper, that delay between consecutive paths is equal to one symbol duration. Subsequently, the data stream is subdivided into blocks and one block is composed of N modulated symbols. Let denote the time domain representation of the i-th transmitted block of size N as $\mathbf{x}^{(i)} = [x_0^{(i)}, \cdots, x_{N-1}^{(i)}]^T$. During the transmission, blocks are serialized and transmitted as a data stream described by the expression

$$\mathbf{x} = [\mathbf{x}^{(0)}, \cdots, \mathbf{x}^{(N_d-1)}]^T \tag{6}$$

where N_d represents the number of blocks per transmitted frame.

The i-th received signal of size N, $\mathbf{r}^{(i)} = [r_0^{(i)}, \cdots, r_{N-1}^{(i)}]^T$ is expressed as

$$\mathbf{r}^{(i)} = \mathbf{H}_0\mathbf{x}^{(i)} + \mathbf{H}_1\mathbf{x}^{(i-1)} + \mathbf{e}^{(i)} \tag{7}$$

where $\mathbf{e}^{(i)} = [e_0^{(i)}, \cdots, e_{N-1}^{(i)}]^T$ denotes the additive white Gaussian noise (AWGN) with 0-mean and a variance of σ_n^2, $\mathbf{H}_0$ and $\mathbf{H}_1$ denote the $N \times N$ channel matrices defined as

$$\mathbf{H}_0 = \begin{bmatrix} \alpha_0 & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & & & \vdots \\ \alpha_{P-1} & & \ddots & \ddots & & \vdots \\ 0 & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & 0 \\ 0 & \cdots & 0 & \alpha_{P-1} & \cdots & \alpha_0 \end{bmatrix} \tag{8}$$

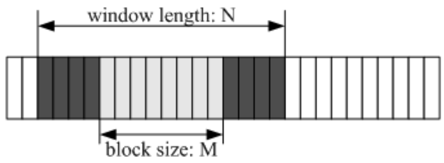

Fig. 1. Time domain representation of the block and the window length definitions

and

$$\mathbf{H}_1 = \begin{bmatrix} 0 & \cdots & 0 & \alpha_{P-1} & \cdots & \alpha_1 \\ \vdots & & & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & \alpha_{P-1} \\ \vdots & & & & & 0 \\ \vdots & & & & & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix} \tag{9}$$

where α_p represents the complex amplitude of the p-th path. In addition, in the rest of the paper, we express the cyclic channel representation as

$$\mathbf{H}_c = \mathbf{H}_0 + \mathbf{H}_1 \tag{10}$$

Due to the cyclic property, we can re-write the expression of the channel representation as

$$\mathbf{H}_c = \mathbf{W}^H \mathbf{\Lambda} \mathbf{W} \tag{11}$$

where $\mathbf{W}$ is a DFT matrix of size $N \times N$, whose (p,q) element is $1/\sqrt{N}.e^{-j\frac{2\pi p.q}{N}}$ and $\mathbf{\Lambda}$ is the diagonal matrix representing the channel decomposition in the frequency domain.

B. SC overlap FDE Receiver and Equalization

Overlap FDE performs receiver filtering in the frequency basis to minimize time domain inter-symbol interference. Its function is the same as that of the time domain equalization. However, for channels with severe delay spread, linear frequency based equalization is simpler in term of computational complexity because equalization is performed by block data. To perform linear channel compensation (e.g. to reduce the complexity of the equalization), we assume that channel equalization is based on the channel representation defined in Eq. (11). This assumption has been validate in [12]-[9]. Four main operations are necessary to perform overlap FDE. First, FFT is performed on a N consecutive elements of the received signal. Then, linear channel compensation in the associated frequency domain is performed. Third step consists of implementing IFFT on the sequence composed by the output of the linear channel compensation. Final stage of the overlap FDE is the selection of the elements that have been limited-affected by the assumption on the linear channel compensation. In [5]-[9], authors propose a selection based on the central part of the processed sequence.

Let first describe the window size and the block size respectively denoted M and N as presented in Fig. 1. In our study, the window size, N ($N \geq M$) is also the size of the FFT and IFFT and the computation of the equalization.

Considering the transmit sequence without any GI and the overlap FDE equalization, the output of the equalization can be described by

$$\mathbf{y}^{(i)} = \mathbf{V}^{N\to M}\mathbf{W}^H\boldsymbol{\Gamma}\mathbf{W}\mathbf{r}^{(i)} \tag{12}$$

where the block size extraction, defined in [6]-[9] is expressed as

$$\mathbf{V}^{N\to M} = diag_N(\underbrace{0,\cdots,0}_{(N-M)/2},\underbrace{1,\cdots,1}_{M},\underbrace{0,\cdots,0}_{(N-M)/2}), \tag{13}$$

where $diag_N(.)$ is the diagonal matrix representation of size N and $\boldsymbol{\Gamma}$ is the matrix representation of the conventional linear equalization in the N-point frequency domain. The frequency domain channel linear equalization (e.g. zero forcing (ZF) or minimum mean square error (MMSE)), based on the circular convolution property, can be described by

$$\boldsymbol{\Gamma} = \begin{cases} (\boldsymbol{\Lambda}^H\boldsymbol{\Lambda})^{-1}\boldsymbol{\Lambda}^H & \text{for ZF} \\ (\boldsymbol{\Lambda}^H\boldsymbol{\Lambda} + \sigma_n^2\mathbf{I}_N)^{-1}\boldsymbol{\Lambda}^H & \text{for MMSE,} \end{cases} \tag{14}$$

where I_N is the identity matrix of dimension N.

III. Estimation of the SINR at the output of FDE

A. Signal and interference representation

In this paper, we design a systematic approach, based on the SINR at the output of equalization scheme, to select of the elements that are interference limited for a fixed window size. It has been shown for SC signals with frequency domain based equalization [12]. The multipath channel in the frequency domain representation mainly affects some specific parts of the signal at the head and tail parts of the frequency representation of the frame. In other words, the effects of the ISI and ICI due to the lost of cyclic convolution property are mainly visible at the two extreme parts of the signal in its frequency domain representation. The output of the FDE can be expressed as

$$\mathbf{z}^{(i)} = \mathbf{W}^H\boldsymbol{\Gamma}\mathbf{W}\mathbf{r}^{(i)} \tag{15}$$

Using Eq.(7) into Eq.(15), we obtain

$$\begin{aligned}\mathbf{z}^{(i)} &= (\mathbf{W}^H\boldsymbol{\Gamma}\boldsymbol{\Lambda}\mathbf{W} - \mathbf{W}^H\boldsymbol{\Gamma}\mathbf{W}\mathbf{H}_1)\mathbf{x}^{(i)} \\ &+ \mathbf{W}^H\boldsymbol{\Gamma}\mathbf{W}\mathbf{H}_1\mathbf{x}^{(i-1)} + \mathbf{W}^H\boldsymbol{\Gamma}\mathbf{W}\mathbf{e}^{(i)}.\end{aligned} \tag{16}$$

By posing

$$\boldsymbol{\Theta} = \mathbf{W}^H\boldsymbol{\Gamma}\boldsymbol{\Lambda}\mathbf{W} - \mathbf{W}^H\boldsymbol{\Gamma}\mathbf{W}\mathbf{H}_1, \tag{17}$$

$$\boldsymbol{\Psi} = \mathbf{W}^H\boldsymbol{\Gamma}\mathbf{W}\mathbf{H}_1, \tag{18}$$

$$\boldsymbol{\Omega} = \mathbf{W}^H\boldsymbol{\Gamma}\mathbf{W}. \tag{19}$$

Eq.(16) can be re-written as

$$\mathbf{z}^{(i)} = \boldsymbol{\Theta}\mathbf{x}^{(i)} + \boldsymbol{\Psi}\mathbf{x}^{(i-1)} + \boldsymbol{\Omega}\mathbf{e}^{(i)}. \tag{20}$$

Based on the signal representation described above, we propose to evaluate the SINR of the v-th element at the output of the FDE

$$\beta_v = \frac{E\Big[|[\boldsymbol{\Theta}\mathbf{x}^{(i)}]_v|^2\Big]}{E\Big[|[\boldsymbol{\Psi}\mathbf{x}^{(i-1)} + \boldsymbol{\Omega}\mathbf{e}^{(i)}]_v|^2\Big]}, \tag{21}$$

where $[.]_v$, $|.|$ and $E[.]$ respectively denote the v-th element of a vector, the norm and the average value.

While the data are independent, the SINR expression can be expressed as

$$\beta_v = \frac{P_s\cdot|\sum_{u=0}^{N-1}\Theta_{v,u}|^2}{P_s\cdot|\sum_{u=0}^{N-1}\Psi_{v,u}|^2 + \sigma_n^2\cdot|\sum_{u=0}^{N-1}\Omega_{v,u}|^2} \tag{22}$$

where P_s is the transmit power per data symbol and $\Theta_{v,u}$, $\Psi_{v,u}$ and $\Omega_{v,u}$ are respectively the (v,u)-th element of $\boldsymbol{\Theta}$, $\boldsymbol{\Psi}$ and $\boldsymbol{\Omega}$.

B. Zero Forcing Detection Case

For the specific case of ZF detection and a constant delay equal to one data sample, the expression of the SINR can be re-written into:

$$\beta_v = \frac{|\sum_{k=0}^{N-1}\frac{a_k - b_{v,k}}{a_v}|^2}{|\sum_{k=0}^{N-1}\frac{b_{v,k}}{a_v}|^2 + \frac{\sigma_n^2}{P_s}\cdot|\sum_{u=0}^{N-1}\Omega_{v,u}|^2} \tag{23}$$

with

$$a_k = \sqrt{N}\cdot\sum_{i=0}^{P-1}\alpha_i\cdot e^{-j\frac{2\pi\cdot k\cdot i}{N}} \tag{24}$$

and

$$b_{v,k} = \sum_{c=N-P+1}^{N-1}\sum_{k'=0}^{c-N+P-1}\alpha_{N-b+k}\cdot e^{-j\frac{2\pi\cdot[v+k']_N\cdot k}{N}} \tag{25}$$

Therefore, the variation of the SINR expression simply depends on the value of $b_{v,k}$. When $b_{v,k}$ tends to 0, the SINR value tends to $\frac{P_s}{\sigma_n^2}$. When $b_{v,k}$ tends to the value of a_k, then the SINR value tends to 0. In addition, the expression of the SINR simply depends on the channel response, the variance of the noise and the initial transmit power.

IV. Adaptive Design of the SC overlap FDE with Fixed Sliding Window Length

A. Proposed procedure

The scheme consists in first estimating the original transmit SNR from the pilot sequence. Then, a Fourier transform is performed for the fixed window length followed by an estimation of the SINR for each component of the orthogonal basis. Then, it consists in evaluating the block size that is associated to the window length, the channel variations in function of the transmission request, the original transmit SNR, based on the estimation of the SINR. Once the window length and

TABLE I

PROPOSED ALGORITHM

Initialization
$M_{max} = argmax(\beta_m)$
$\beta_{max} = \beta_{m_{max}}$

Upper bound of the block
$M_{sup} = M_{max}$
While($\beta_{M_{sup}+1} \geq \xi \cdot \beta_{max}$)
then M_{sup}++

Lower bound of the block
$M_{inf} = M_{max}$
While($\beta_{M_{inf}-1} \geq \xi \cdot \beta_{max}$)
then $M_{inf}-$

Final step
$M_{opt} = M_{sup} - M_{inf}$
$\mathbf{V}^{N\to M} = diag_N(\underbrace{0, \cdots, 0}_{M_{inf}}, \underbrace{1, \cdots, 1}_{M_{opt}}, \underbrace{0, \cdots, 0}_{N-M_{sup}})$

the associated block size are determined, the overlap FDE equalization is performed by simply sliding the window by the size of the associated block. The effect of the Fourier transforms is to reject the interferences to the extreme parts of the associated window. Basically, overlap FDE includes this specificity in the way the received signal is equalized. Let $\boldsymbol{\beta}$ denotes a vector representation of β_m, the SINR associated to the window size N, as $\boldsymbol{\beta} = [\beta_0, \cdots, \beta_{N-1}]^T$.

First, at the initialization step, the element in the window of size N with the highest SINR is identified, denoted β_{max}. We also define M_{max} the index of the identified element. Then, evaluation of the block size is performed through two stages. The first stage is related to the identification of the upper bound of the block size. The procedure consists of evaluating the longest consecutive sequence of elements that verifies the constraint about the minimum required SINR denoted $\xi \cdot \beta_{max}$ with ξ the acceptable SINR variation. Similarly, the lower bound of the block is estimated by selecting the longest sequence which verifies the constraint described by $\beta_m \geq \xi \cdot \beta_{max}$ with $0 \leq m < M_{max}$ and has to finish by the element M_{max}.

Complete algorithm to systematically estimate the optimum block size in function of the window size, the received SINR and the channel representation is described in Table I.

B. Complexity Evaluation

Given a window size of N and the block size of M equals to $(2N \cdot log(N) + N)$, we consider a transmit frame of length $N_d \cdot N$, and for simplicity we assume that $(N_d \cdot N)/M$ is a integer value. The complexity of the overlap FDE system is equivalent to the total number of operations to process the frame at the received part which is simply equal to $(2N \cdot log(N) + N) \cdot (N_d \cdot N)/M$.

TABLE II

SIMULATION PARAMETERS

Carrier frequency	2.4 GHz
Bandwidth	20 MHz
Modulation scheme	QPSK, 16-QAM, 64-QAM
Channel encoder	No code, convolutional codes
Channel estimation	Perfect CSI
Sample period	0.05μs
Number of data packets	30

TABLE III

SIMULATION PARAMETERS: CHANNEL MODEL

Number of paths	10
Sample period between consecutive paths	0.05μs
Maximum delay spread	0.45μs

V. NUMERICAL SIMULATION

We now evaluate the performance of the proposed method to adapt the block size in a mutli-path fading environment. The main simulation parameters are summarized in Table II. For all simulations, a multi-path model is assumed and the carrier frequency is equal to 2.4GHz. For the conventional SC-FDE with (or without) appropriate GI insertion and the IFFT/FFT size is set to 64 or 128 points.

To verify the behavior of the equalization and selection method based on the estimation of the SINR at the output of the FDE, we propose to evaluate the performance for the channel environment described in Table III and for uncoded and coded systems.

For the simulation, we propose to associate the range of SINR with the block size by $\xi = (1 - \Gamma_{\%})$, where $\Gamma_{\%}$ is the range of acceptable variation of the SINR.

Figs. 2 and 3 illustrate the BER performances of the proposed scheme for the adaptive window size selection for coded system and 2 different modulations (QPSK and 64-QAM). Simulation results are plotted for different value of $\Gamma_{\%}$. Similarly to the uncoded case, impact of the reliability of the system performance directly depends on the size of the window.

TABLE IV

IMPACT OF SINR RANGE ON THE BLOCK SIZE FOR QPSK MODULATION AND R=1/2

Modulation	Coding Rate	$\Gamma_{\%}$	Block size
QPSK	1/2	95%	28
QPSK	1/2	90%	33
QPSK	1/2	85%	37
QPSK	1/2	70%	42

In addition, Tables IV and V show the window size value for a block size equal to 64.

VI. CONCLUSION

This paper introduces a systematic design of overlap FDE for SC transmission. Based on the analysis of SINR,

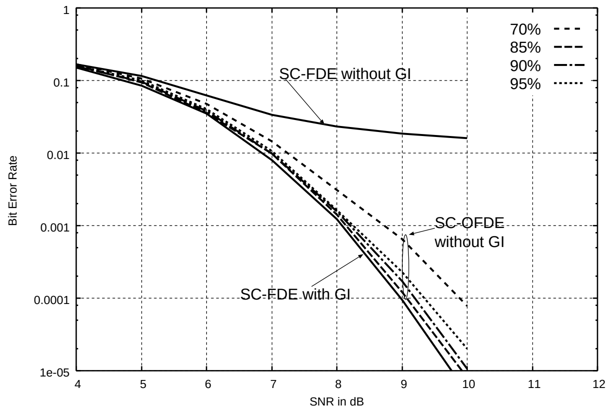

Fig. 2. BER performance for QPSK modulation, R=1/2 and N=64

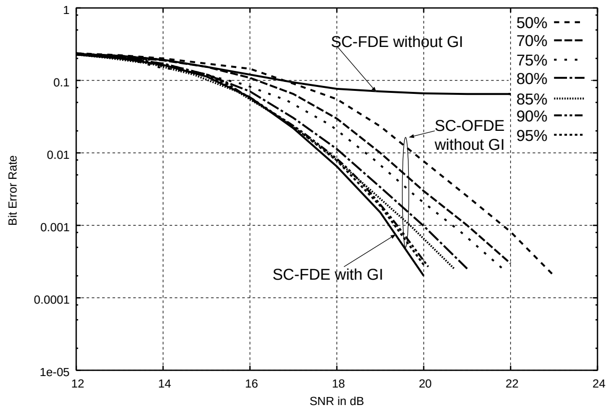

Fig. 3. BER performance for 64-QAM modulation, R=1/2 and N=64

an adaptive adjustment of the overlap FDE block size is proposed. The proposed method consists of evaluating the SINR of each symbol at the FDE output and adjusting the block size in the overlap FDE depending on the SINR. In addition, we propose to adjust the block size in order to control the computational complexity of the equalization per processed sample associating with the average bit error rate (BER) of the system. The simulation results have validated the proposed method in terms of BER performance for QPSK and QAM modulations and coded systems. Moreover, the proposed method can be easily extend to any other advanced equalization for SC overlap FDE transmission and any powerful channel encoder such as the turbo codes [18] or the low density parity check codes [19]-[20].

TABLE V
IMPACT OF SINR RANGE ON THE BLOCK SIZE FOR 64-QAM MODULATION AND R=1/2

Modulation	Coding Rate	$\Gamma_{\%}$	Block size
64-QAM	1/2	95%	19
64-QAM	1/2	80%	22
64-QAM	1/2	70%	25
64-QAM	1/2	50%	32

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