Power Allocation Scheme for Coded MIMO Multi-Carrier Systems

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***Abstract*— Orthogonal Frequency Division Multiplexing (OFDM) is a popular method for high data rate wireless transmission. Combining OFDM with antenna arrays at the transmitter and receiver enhances the capacity gain. This paper proposes a novel transmit power allocation for Coded Multi-Input Multi-Output Multi-Carrier Modulation (MIMO-MCM). This technique consists of adapting the power allocation in the spatial domain (i.e. through the different transmit antennas) in term of the channel conditions by the optimization process based on the optimality of the global Bit Error Rate (BER). Simulation results show significant gains for several antenna configurations.**

***Keywords-* OFDM, MIMO, Lagrangian method, global BER optimization.**

I. Introduction

High data rate wireless systems with very small symbol periods usually face unacceptable Inter Symbol Interference (ISI) originated from multi-path propagation and their inherent delay spread. Orthogonal Frequency Division Multiplexing (OFDM) has emerged as one of the most practical techniques for data communication over frequency-selective fading channels [1]-[2]. In OFDM, the computationally-efficient Fast Fourier Transform (FFT) is used to transmit data in parallel over a large number of orthogonal subcarriers. When an adequate number of subcarriers are with a cyclic prefix, subcarrier orthogonality is maintained even in the presence of frequency selective fading. Orthogonality does not imply any subcarrier interference and permits simple high-performance data detection which improves capacity in the wireless system with high spectral efficiency (bps/Hz). On the other hand, to increase the spectral efficiency of wireless link, Multi-Input Multi-Output (MIMO) systems can be employed to transmit several data streams in parallel at the same time and on the same frequency but from different transmit antennas [3]. However, at the receiver side, multi-stream detection is needed. In this paper, we propose a method to efficiently allocate transmit power in term of the channel conditions, and the estimate of Bit Error Rate (BER) for coded transmit sequence. In Section II, we describe the MIMO OFDM systems and the associated signal model. In Section III, we introduce the estimate of the BER for channel encoded sequence and we describe the proposed power allocation scheme. Section IV gives simulation results obtained through frequency selective fading channels over coded QPSK and QAM modulations and conclusions are drawn in Section V.

II. MIMO OFDM signal

A. MIMO OFDM transmitter

We consider a MIMO OFDM system with *Nt* transmit (TX) and *Nr* receive (RX) antennas. MIMO OFDM transmitter consists of *Nt* OFDM transmitters among which the incoming bits are multiplexed, then channel coding, interleaving and modulation mapping are performed in parallel. The modulated signals are frequency-division multiplexed by *N*-point Inverse Discrete Fourier Transform (IDFT). The resulting OFDM signal is then converted into an analog signal by a digital-analog (D/A) converter with rate *1/Ts* where *Ts* is the sample period, up-converted (U/C) to the RF band and transmitted in the air. For reliable detection, it is typically necessary that the receiver knows the wireless communication channel and keeps track of phase and amplitude variations. To enable the estimation of the wireless communication channel, the transmitter occasionally sends known training symbols. For instance, a preamble, which includes channel training sequences, is added to every packet in IEEE 802.11 WLAN standards [4]-[5]. Consider an OFDM system that transmits *N* modulated data symbols in the *i*-th OFDM symbol period through *N* sub-channels. For the *i*-th block symbol, the transmitted baseband OFDM signal is expressed as [6]-[7]:

$$s_{i,n}^{(p)} = \frac{1}{\sqrt{N}} . \sum_{k=0}^{N-1} x_{i,k}^{(p)} . \exp\left\{ j \frac{2\pi .nk}{N} \right\} \quad (1)$$

where $x_{i,k}^{(p)}$ is the modulated data symbol of the p-th transmit antenna for the i-th OFDM symbol. To combat ISI and Inter Carrier Interference (ICI), Guard Interval (GI) [8] such as Cyclic Prefix (CP) or Zero Padding (ZP) is added to the OFDM symbols. In the case of CP, the last N_g samples of every OFDM symbol are copied and added to the heading part. The transmit signal can be described as follow:

$$\tilde{s}_{i,n}^{(p)} = \begin{cases} s_{i,N-Ng+n}^{(p)}, & \text{for } 0 \le n < N_g \\ s_{i,n-Ng}^{(p)}, & \text{for } N_g \le n < N + N_g \end{cases} \quad (2)$$

B. MIMO OFDM receiver

At the receiver, signals are passed through a down- converter (D/C) and a digital to analog converter with rate *1/Ts*. Then, the GI is removed and the *N*-point Discrete Fourier Transform (DFT) is performed per receiver branch. Since channel parameters suppose to be frequency selective, MIMO detection has to be done per OFDM subcarrier. Therefore, the received signals of subcarrier *i* are routed to the *i*-th MIMO detector to recover the *Nt* transmitted data signals per subcarrier. Finally, de-mapping, de-interleaving and channel decoding are performed for the *Nt* parallel streams and the resulting data are combined to obtain the original binary sequence.

C. Channel representation and detection scheme

We assume that the system is operating in a frequency selective Rayleigh fading environment [9] and the communication channel remains constant during a packet transmission. Data frame duration is assumed to transmit within the coherent time of the wireless system. In this case, channel variations remain constant during one frame transmission and may change between consecutive frame transmissions. We suppose that the fading channel can be modeled by a discrete-time baseband equivalent $(L-1)$-th order finite impulse response (FIR) filter where L represents time samples corresponding to the maximum delay spread. In addition, an Additive White Gaussian Noise (AWGN) with N_r independent and identically distributed (iid) zero mean, complex Gaussian elements is assumed. When the maximum delay spread does not exceed GI, since ISI does not occur on MIMO OFDM symbol basis, the frequency domain MIMO OFDM signal after removal of GI is described by:

$$y_{j,m}^{(q)} = \sum_{p=0}^{N_t-1} h_m^{(q,p)}.x_{j,m}^{(p)} + n_{j,m}^{(q)} \tag{3}$$

where $y_{j,m}^{(q)}$ is the received signal at the q-th received antenna for the j-th OFDM symbol and the m-th sub-carrier and $h_{j,m}^{(q,p)}$ is the channel parameter from the p-th transmitting antenna to the q-th receiving antenna which composes the MIMO channel matrix. In addition, $n_{j,m}^{(q)}$ denotes the AWGN for the q-th received antenna. Thus it results in a frequency-flat-fading signal model per sub-carrier. For simplicity, without losing any generality, we will omit writing the index for both the sub-carrier and the symbol indicators. Hereafter, the received signal can simply be written as:

$$\mathbf{Y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{4}$$

where the (q,p)-th element of $\mathbf{H}$ is $h^{q,p}$, $\mathbf{Y}$ denotes the received vector, $\mathbf{X}$ represents the transmitted signal, and $\mathbf{n}$ represents zero mean, complex AWGN with covariance matrix equals to:

$$\mathbf{E}[\mathbf{n}\mathbf{n}^H] = \sigma_n^2 \mathbf{I}_{N_r} \tag{5}$$

The zero forcing (ZF) criteria [10] is used for simulation. We denote $\mathbf{G} = \{\mathbf{G}_m^{(l,n)}\}_{\forall l,n,m}$ the matrix representation of the detection scheme. In a ZF linear detector, the received signal vector is multiplied with a filter matrix which is a pseudo inverse of the channel response

$$\mathbf{G} = (\mathbf{H}^H\mathbf{H})^{-1}.\mathbf{H}^H \tag{6}$$

where we assume $N_r \geq N_t$.

III. Power Allocation Scheme

A. Bit Error Rate approximation

We next list the assumptions adopted in this paper. The channel is frequency flat, and remains invariant per frame, but is allowed to vary from frame to frame. This corresponds to a block fading channel model, which is suitable for slowly-varying channels [11]. As a consequence, power allocation is adjusted on a frame-by-frame basis. Perfect channel state information (CSI) is available at the receiver. The corresponding power selection is fed back to the transmitter without any error and latency.

The assumption that the feedback channel is error free and has no latency, could be at least approximatively satisfied by using a fast feedback link with powerful error control for feedback information. Further considerations on system design with delayed or noisy CSI will be in the future considered.

For flat fading channel adhering the previous assumptions, the channel characteristics is captured by the received SNR. The received SNR is a function of the variance of the noise, the channel state information, and the transmit power. Since the channel varies from frame by frame, the approximated BER can be described as a function of the received SNR (i.e. transmit power for each transmit antenna and channel condition). In [12], a general formula describing this relation in a flat fading channel environment is proposed. Typically, the BER for any modulation and coding rate can be simply estimated as follows:

$$f(\beta_{l,m}, p_{l,m}) \approx a \times \exp\big\{ - b \times \beta_{l,m} \times p_{l,m} \big\} \tag{7}$$

with:

$$\beta_{l,m} = \frac{1}{(2^{N_m}-1).\sigma_n^2.\sum_{n=0}^{N_r-1} |\mathbf{G}_m^{(l,n)}|^2} \tag{8}$$

where N_m is the number of bits per symbol.

In (7), $p_{l,m}$ represents the allocated power on the l-th transmit antenna on the m-th subcarrier.

The implementation of the convolutional code is based on the description proposed in [4]. The generator polynomial of the mother code is $g = [133, 171]$. The coding rates are obtained from the puncturing pattern described in [4].

Tables I, II and III summarize the heuristic value of the parameters *a* and *b* defined in (7) for several coding rates and several modulations which are obtained by fitting (7) to simulated data.

TABLE I

TRANSMISSION MODES FOR QPSK MODULATION

Modulation	QPSK	QPSK
Coding Rate	1/2	3/4
Rate (bits/symb.)	1	1.5
a	7	16
b	9.5	5.4

TABLE II

TRANSMISSION MODES FOR 16-QAM MODULATION

Modulation	16-QAM	16-QAM
Coding Rate	1/2	3/4
Rate (bits/symb.)	2	3
a	4	14
b	11	6

B. Proposed principle

The basic principle of the spatial domain power allocation for MIMO OFDM signal is to perform spatial domain optimization of the transmit power in terms of the channel state information (CSI) and the expression of the global BER for the different modulations and coding gains. By simply performing transmit power allocation from the estimated value of the BER, we propose to optimize the transmit power allocation by performing Lagrangian method on the BER expression. Constraint is added in order to keep constant the global transmit power at the transmitting part.

C. Mathematical description

Proposed power allocation consists of optimizing the BER function described below by estimating optimum power allocation [13]. Power allocation algorithm determines how to allocate the power in the space domain. The global power resource assigned by the proposed algorithm satisfies the simple relation that the total transmit power is kept constant. This constraint can be described by:

$$\sum_{l=0}^{N_t-1} p_{l,m} = N_t.\overline{P}_m \tag{9}$$

where $\overline{P}_m$ denotes the average Transmit Power on the subcarrier *m*.

The average BER becomes minimal when the BER is minimized for each given channel state. Equivalent mathematical representation can be given by:

$$\begin{cases} \min f(\beta_{l,m}, p_{l,m}) \\ \\ \text{subject to:} \\ \sum_{l=0}^{N_t-1} p_{l,m} = N_t.\overline{P}_m \end{cases} \tag{10}$$

One possibility to solve this optimization problem is to apply the Lagrangian procedure described as:

$$J(p_{0,m}, ..., p_{N_t-1,m}) = \frac{1}{N_t} \sum_{l=0}^{N_t-1} f(\beta_{l,m}, p_{l,m}) + \lambda \times (\sum_{l=0}^{N_t-1} p_{l,m} - N_t \times \overline{P}_m) \tag{11}$$

Then, optimal solutions are obtained by solving for each transmit antenna:

$$\begin{cases} \frac{1}{N_t}.\frac{\partial}{\partial p_{l,m}}\left(\sum_{l=0}^{N_t-1} f(\beta_{l,m}, p_{l,m})\right) + \lambda = 0 \\ \sum_{l=0}^{N_t-1} p_{l,m} - N_t.\overline{P}_m = 0 \end{cases} \tag{12}$$

By introducing the explicit estimation of the BER in the set of equations, we can write for each transmit antenna that:

$$\begin{cases} \frac{-a.b.\beta_{l,m}}{N_t} \times \exp(-b.\beta_{l,m}.p_{l,m}) + \lambda = 0 \\ \sum_{l=0}^{N_t-1} p_{l,m} - N_t.\overline{P}_m = 0 \end{cases} \tag{13}$$

After calculation and rearrangement, we finally obtain the following general solution:

$$p_{l,m} = \left[1 + \sum_{\substack{u=0 \\ u\neq l}}^{N_t-1} \frac{\beta_{l,m}}{\beta_{u,m}}\right]^{-1} \times \left[N_t.\overline{P}_m + \frac{1}{b} \times \sum_{\substack{u=0 \\ u\neq l}}^{N_t-1} \frac{1}{\beta_{u,m}} \times \log\left(\frac{\beta_{l,m}}{\beta_{u,m}}\right)\right] \tag{14}$$

Due to the nature of the Lagrangian calculation (only mathematical solutions are obtained), we need to add a constraint when the output of the Lagrangian optimization does not reflect any physical solution, typically when we obtain: $p_{l,m} \leq 0$. In this case we propose to apply the conventional scheme, such as equal power allocation.

TABLE III

TRANSMISSION MODES FOR 64-QAM MODULATION

Modulation	64-QAM	64-QAM
Coding Rate	1/2	3/4
Rate (bits/symb.)	3	4.5
a	1.5	7
b	12	6

IV. EXPERIMENTATION

We now evaluate the performance of the proposed power allocation method for MIMO-OFDM scheme in a multi-path fading environment. Main simulation parameters are shown in Table IV. We assume perfect knowledge of the channel conditions both at the transmitting and receiving parts. For all simulations, a multi-path model exponential with 1-dB decay is assumed. Main parameters are roughly based on the IEEE 802.11g standard; carrier frequency is equal to 2.4GHz, the IFFT/FFT size is 64 points and the guard interval is set

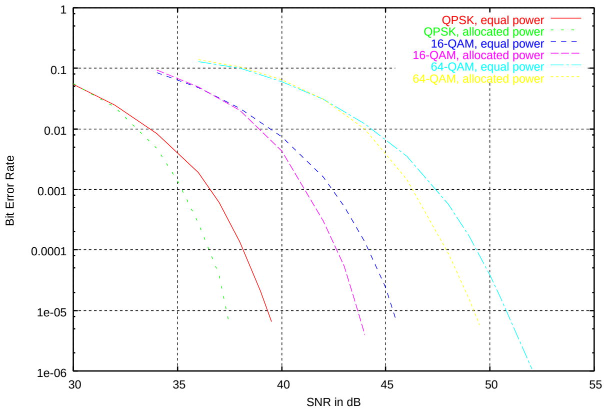

Fig. 1. Performance Simulation for Nt=Nr=4, R=1/2

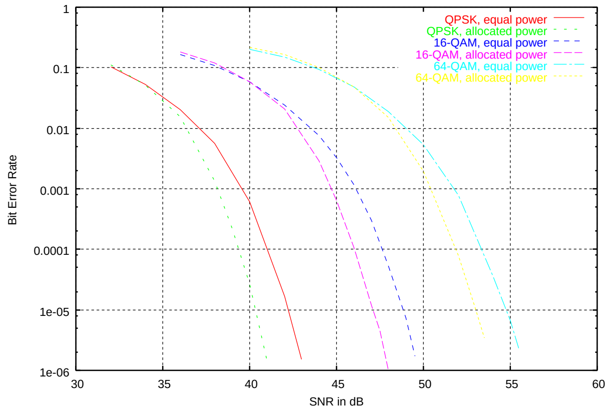

Fig. 2. Performance Simulation for Nt=Nr=4, R=3/4

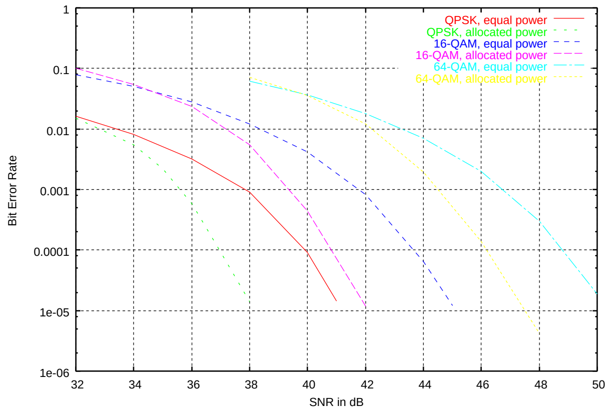

Fig. 3. Performance Simulation for Nt=Nr=8, R=1/2

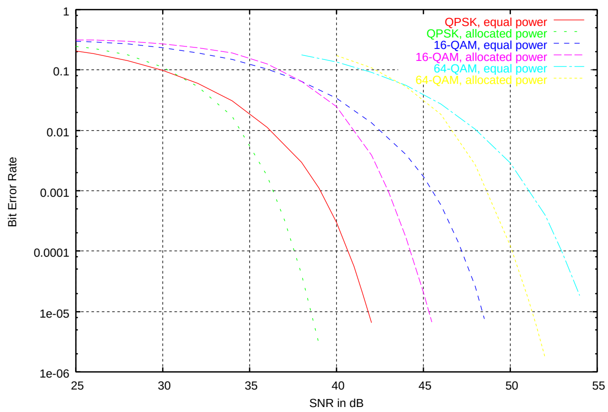

Fig. 4. Performance Simulation for Nr=Nt=8, R=3/4

up at 16 samples. As described in section III, standardized convolutional code and punctured function are also detailed in the WLAN standard. Effect of proposed scheme for several number of transmit and receive antennas is highlighted. In addition, we show the impact of the coding gain on the power allocation method.

TABLE IV
SIMULATION PARAMETERS

Carrier Frequency	2.4 GHz
Bandwidth	20 MHz
Modulations	QPSK, 16-QAM, 64-QAM
Channel encoder	Convolutional code
Channel estimation	Perfect CSI
Number of data subcarrier	64
Guard Interval length	16
Channel model	5-path, Rayleigh Fading
Sample period	$0.05\mu s$
Number of data packet	50
(*Nt*,*Nr*) configuration	(4,4) and (8,8)

Figs.1, 2, 3 and 4 show the Bit Error Rate (BER) versus the average total received SNR (dB) for different antenna configurations, several modulation schemes and two specific coding rates (R=$1/2$ and R=$3/4$). Both conventional and proposed schemes are plotted in these figures and results are presented for ZF detection scheme.

In Fig. 1, the benefit of performing the proposed scheme is highlighted for the case of $N_t = N_r = 4$ and R=$1/2$ coding rate. Simulation results show that at average BER=10^{-5}, 1.8dB gain is obtained in the case of QPSK modulation. Gain becomes equal to 1.6dB at BER=10^{-5}, for both 16-QAM and 64-QAM modulations. For low average total received SNR, at BER=5.10^{-2} proposed and conventional schemes have similar performance for the three modulation schemes. This effect comes from the condition that only realistic solutions are kept into consideration and for relatively low total SNR, proposed scheme is not used (i.e. no solution is available to allocate specific power on each transmit antenna). Gain of the proposed method becomes to be significant from the average BER=5.10^{-3}.

In Fig. 2, results are presented for the case of $N_t = N_r = 4$ and the coding rate R=$3/4$. The gain obtained by performing proposed power allocation scheme becomes equal to 3dB for QPSK modulation and 2.8dB for QAM modulations at average BER=10^{-5}. For the specific coding rate R=$3/4$, the proposed scheme is more efficient than the equal power allocation from the average BER=3.10^{-2} for the three types of modulation.

In Fig. 3, results for the case of $N_t = N_r = 8$ and coding rate R=$1/2$ are presented. The gain obtained by performing proposed power allocation scheme becomes relatively significant. At average BER=10^{-5}, 3.0dB gain is obtained for QPSK modulation and gain becomes equal to 2.9dB for 16-QAM and 64-QAM modulations.

Average BER versus the total SNR is presented in the Fig. 4 for the case of $N_t = N_r = 8$ and coding rate R=$1/2$. At average BER=10^{-5}, 3.5dB gain is obtained for QPSK modulation and the gain becomes equal to 3.2dB for QAM modulations. For low average total received SNR, at BER=10^{-1} conventional schemes outperforms the proposed method. Proposed method becomes efficient from the average BER=5.10^{-3}.

V. Conclusion

In this paper, a novel power allocation scheme for MIMO based transmission has been presented. The basic procedure consists of allocating transmit power in terms of the channel conditions and the global BER requirement. The simulation results have shown promising results in term of BER for several sets of antenna configuration, different modulations and two different coding gains. Extension of the proposed power allocation scheme can be easily proposed to no-linear detection scheme such as the BLAST solution [14] or the Maximum Likelihood Detection (MLD) as in [15]. Finally, results including channel encoding have been limited to convolutional but extension to any other powerful encoder like the Turbo Codes (TC) [16] or Low Density Parity Check (LDPC) [17]-[18] can also be included in the modulated transmission. Finally, future orientation for this work would include the introduction of the error in the channel estimation [19]. For all these different modifications, the corresponding values of *a* and *b* in (7) must be updated.

Acknowledgment

The authors would like to thank Mr. Haruo Mio of FT R&D Tokyo and Dr. Patrick Tortelier FT R&D for their valuable discussions.

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