Transmit Beamforming and Power Allocation for Downlink OFDMA Systems

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***Abstract*—This paper considers a transmit beamforming and subcarrier power allocation method for orthogonal frequency division multiple access (OFDMA) systems. As the beamforming vector control criterion, we employ the maximization of so-called signal-to-leakage-plus-noise ratio (SLNR) at each base station, which enable us to obtain closed form beamforming vector by using only locally available information. We also discuss the local optimality of the SLNR based beamforming vector. As for the subcarrier power allocation, two different approaches are employed, namely, the equalization of signal-to-interference-plus-noise ratio (SINR) for subcarriers and the maximization of sum rate of subcarriers. Computer simulation results show the validity of the transmit beamforming and power allocation method with highlighting the difference between the two power allocation algorithms.**

***Index Terms*—OFDMA, transmit beamforming, power allocation, iterative water-filling**

I. Introduction

Block transmission schemes, such as orthogonal frequency division multiplexing (OFDM), have been widely applied to various high speed wireless communications systems as well as TV broadcasting systems [1] because of the inherent robustness to frequency selective fading channels attributed to the effective and efficient frequency domain equalization using cyclic prefix [2]–[4]. Moreover, much effort to apply block transmission schemes to mobile communications systems has been made as typified by WiMAX system, where orthogonal frequency division multiple access (OFDMA) [5] is adopted for the physical layer / medium access control layer protocol.

One of the serious problems in cellular mobile communications systems is how to cope with the co-channel interference from other cells, especially when frequency reuse factor is set to be 1 in order to achieve high spectral efficiency. Receiving beamforming using multiple antenna elements with the combining vector based on maximum signal-to-interference-plus-noise ratio (SINR) criterion is effective for the interference suppression. However, employing multiple antenna elements and calculating the vector at the mobile terminal are undesirable from view points of cost and power consumption. Therefore, for downlink communications in the cellular systems, transmit beamforming is more suitable.

In this paper, we consider the problem of downlink transmit beamforming for OFDMA systems. Unlike the receiving beamforming, the determination of the beamforming vector based on the SINR is difficult problem. This is not only because the received SINR of each user becomes a function of beamforming vectors of all the base stations, but also because SINRs at all the mobile terminals have to be taken into consideration for the calculation of the vector. Instead of the simultaneous maximization of received SINRs, we utilize so-called *signal-to-leakage-plus-noise ratio (SLNR)* [6] as the optimization metric of beamforming vectors. The same idea as the SLNR is also used in [7], where the beamforming vector is determined by using the received SINR of *virtual uplink*. The SLNR of the base station is defined as the ratio of the received signal power from the base station at the desired mobile terminal to the received signal power at the other terminals plus noise power. With the SLNR based criterion, each base station can obtain closed form expression based only on locally available information. We discuss the validity of the criterion using the SLNR in terms of the local optimality of the beamforming vector. We also consider the power allocation problem over subcarriers combined with the transmit beamforming in order to further reduce the impact of the co-channel interference.

As the power allocation strategy, we take two different approaches; the equalization of SINR for all the subcarriers and the maximization of sum rate of subcarriers. The former is similar to the method proposed in [7] or [8], although we impose the constraint on the total transmit power. The latter is based on the idea of *iterative water-filling* [9]-[11], and each user tries to maximize his own sum rate of all the subcarriers in a distributed manner. From the computer simulation results, we discuss achievable rate of the SLNR based beamforming vector comparing with the performance of maximum-ratio-combining (MRC) weight, zero-forcing (ZF) weight, and their linear combinations [12], [13]. Moreover, the performance of the transmit beamforming and power allocation method is evaluated with highlighting the difference between the two power allocation algorithms.

II. Signal Model

Consider downlink of OFDMA systems with N base stations and N mobile terminals. For each of the base stations, one mobile terminal out of the N mobile terminals is a desired terminal in the cell of the base station, while the signals from the rest of the $N-1$ base stations are considered as interference for the mobile terminal. Let $\mathbf{s}_i = [s_i^0, \cdots, s_i^{M-1}]^{\mathrm{T}}$ denote the frequency domain transmitted signal block from the i-th base station to the i-th mobile terminal, where M is the number of subcarriers and s_i^m is the symbol on the m-th subcarrier of the OFDMA signal. Moreover, P_i^m denotes the transmitted signal power of the i-th base station on the m-th subcarrier, and we define $\mathbf{w}_i^m = [w_i^{m,1}, \cdots, w_i^{m,Q}]^{\mathrm{T}}$ as the transmit beamforming weight vector on the m-th subcarrier of the transmitted signal from the i-th base station, where Q is the number of antenna elements at each base station. Furthermore, assuming the length of the guard interval is greater than or equal to the order of the channel impulse response between the q-th antenna element of the i-th base station and the j-th mobile terminal $\mathbf{h}_{ji}^q = [h_{ji}^q(0), \cdots, h_{ji}^q(L-1)]^{\mathrm{T}}$, the frequency response between the q-th antenna element of the i-th base station and the j-th mobile terminal is given by

$$\begin{bmatrix} \lambda_{ji}^{0,q} \\ \vdots \\ \lambda_{ji}^{M-1,q} \end{bmatrix} = \mathbf{D} \begin{bmatrix} \mathbf{h}_{ji}^q \\ \mathbf{0}_{(M-L)\times 1} \end{bmatrix}, \tag{1}$$

where $\mathbf{D}$ is $M \times M$ a discrete Fourier transform (DFT) matrix, whose $\{m, n\}$ element is $\{\mathbf{D}\}_{m,n} = 1/\sqrt{M} \exp(-j\frac{2\pi mn}{M})$ and $\mathbf{0}_{(M-L)\times 1}$ denotes a zero vector of $(M-L) \times 1$. Defining the frequency response vector on the m-th subcarrier as $\boldsymbol{\lambda}_{ji}^m = [\lambda_{ji}^{m,1}, \cdots, \lambda_{ji}^{m,Q}]^{\mathrm{T}}$, the received signal at the j-th mobile terminal on the m-th subcarrier is written as

$$r_j^m = \sum_{i=1}^{N} \sqrt{P_i^m} (\mathbf{w}_i^m)^{\mathrm{H}} \boldsymbol{\lambda}_{ji}^m s_i^m + n_j^m, \tag{2}$$

where n_j^m denotes a zero mean additive white Gaussian noise (AWGN) at the j-th terminal on the m-th subcarrier with the variance of σ_n^2.

For the j-th mobile terminal, only the signal from the j-th base station is the desired signal, and the signals from the other base stations results in the interference. Therefore, the SINR at the j-th terminal on the m-th subcarrier is given by

$$\Gamma_j^m = \frac{P_j^m (\mathbf{w}_j^m)^{\mathrm{H}} \boldsymbol{\lambda}_{jj}^m (\boldsymbol{\lambda}_{jj}^m)^{\mathrm{H}} \mathbf{w}_j^m}{\sum_{\substack{i=1 \\ i \neq j}}^{N} P_i^m (\mathbf{w}_i^m)^{\mathrm{H}} \boldsymbol{\lambda}_{ji}^m (\boldsymbol{\lambda}_{ji}^m)^{\mathrm{H}} \mathbf{w}_i^m + \sigma_n^2}. \tag{3}$$

Since (3) includes beamforming vectors of all the base stations $\mathbf{w}_1^m, \cdots, \mathbf{w}_N^m$ and channel frequency response between the j-th mobile terminal and all the base stations $\boldsymbol{\lambda}_{j1}^m, \cdots, \boldsymbol{\lambda}_{jN}^m$, the maximization of the SINR (3) with respect to beamforming vectors cannot be performed without exchanging information on the vectors or channel responses among base stations even for given transmit power of P_i^m. Moreover, even if we can obtain the beamforming vectors, which maximize (3), the vectors result in poor performance in general for the other mobile terminals. Therefore, unlike the case of the receiving beamforming, SINRs of all the users $\Gamma_1^m, \ldots, \Gamma_N^m$ have to be taken into the consideration for the determination of the beamforming vectors and the power allocation. In order to avoid the complicated joint optimization problem, we take a two-step suboptimal approach for the transmit beamforming and the power allocation; we firstly determine beamforming vectors for a given power allocation (say, uniform power allocation) using a different performance metric from the received SINR, and then perform subcarrier power allocation for the beamforming vectors. In the following sections, we describe details of the transmitted beamforming and power allocation methods.

III. Beamforming Weight Control

We consider the maximization of SLNR [6] at each base station for the beamforming vector determination. The SLNR of the j-th base station is defined as the ratio of the received signal power from the base station at the desired mobile terminal (the j-th terminal) to the received signal power at the other terminals plus noise power. This can be also considered as the ratio obtained from (3) by replacing the interference power observed at the j-th mobile terminal in the denominator with the interference power caused by the j-th base station. The SLNR is written as

$$\tilde{\Gamma}_j^m = \frac{P_j^m (\mathbf{w}_j^m)^{\mathrm{H}} \boldsymbol{\lambda}_{jj}^m (\boldsymbol{\lambda}_{jj}^m)^{\mathrm{H}} \mathbf{w}_j^m}{\sum_{\substack{i=1 \\ i \neq j}}^{N} P_i^m (\mathbf{w}_j^m)^{\mathrm{H}} \boldsymbol{\lambda}_{ij}^m (\boldsymbol{\lambda}_{ij}^m)^{\mathrm{H}} \mathbf{w}_j^m + \sigma_n^2}. \tag{4}$$

Key issue here is that the SLNR is composed only by locally available information, while the SINR (3) includes some values, which are not directly observable for the j-th base station, like $\mathbf{w}_i^m$ or $\boldsymbol{\lambda}_{ji}$ for $i \neq j$.

Each transmit beamforming vector is controlled so that the SLNR of each base station is maximized. For given transmit power $\tilde{P}_i^m$, the beamforming vector, which maximizes (4) is given by

$$(\mathbf{w}_j^m)^{\mathrm{SLNR}} = \left(\sum_{i=1}^{N} P_i^m \boldsymbol{\lambda}_{ij}^m (\boldsymbol{\lambda}_{ij}^m)^{\mathrm{H}} + \sigma_n^2 \mathbf{I}_Q \right)^{-1} \sqrt{P_j^m} \boldsymbol{\lambda}_{jj}^m, \tag{5}$$

where $\mathbf{I}_Q$ denotes $Q \times Q$ identity matrix. In this way, by utilizing the SLNR, we can obtain closed form expression of the beamforming vector. The validity of the utilization of the SLNR based vector is discussed in Sec. V.

IV. Power Allocation

We consider the transmit power allocation for fixed beamforming vectors $\mathbf{w}_j^m$ with total power constraint of

$$P = \sum_{m=0}^{M-1} P_j^m. \tag{6}$$

In this section, we employ two different power allocation algorithms, where the transmit power of each subcarrier is iteratively adjusted based on the observed SINR of the subcarrier at the desired mobile terminal. One approach is to give larger power to the subcarrier with relatively lower SINR, while the other algorithm gives larger power to the subcarrier with relatively better SINR. The former tries to equalize SINRs of subcarriers, and in this sense, the approach is similar to the method proposed in [7] or [8], where the transmit power is minimized under the condition that the received SINR is greater than or equal to a predetermined required SINR. The difference is that we impose the constraint on the total transmit power, while in the method [7] or [8] the total power can be very large depending on channel conditions. It should be noted that the approach in [7] or [8] allows each user to achieve the unique Nash equilibrium, while the solution is inefficient in terms of Pareto optimality [14]. On the other hand, the latter is based on the idea of *iterative water-filling* [9]-[11], and each user tries to maximize his own sum rate of all the subcarriers in a distributed manner. Note that, although the system considered in the paper has multiple antennas, the iterative water-filling is performed not in spatial domain as in [11] but in frequency domain.

Since both algorithms determine the transmit power by an iterative manner, the transmit power of the j-th base station on the m-th subcarrier at the n-th iteration is described as $P_j^m(n)$ hereafter.

A. Power Allocation Algorithm #1

In this algorithm, the transmit power of each subcarrier is adjusted so that the power is inversely proportional to the observed SINR at the desired mobile terminal as

$$P_j^0 : P_j^1 : \cdots : P_j^{M-1} = \frac{1}{\Gamma_j^0} : \frac{1}{\Gamma_j^1} : \cdots : \frac{1}{\Gamma_j^{M-1}}. \quad (7)$$

Then, the recursive update equation of the transmit power can be written as

$$P_j^m(n+1) = \frac{x}{\Gamma_j^m(n)} P_j^m(n), \quad (8)$$

where $\Gamma_j^m(n)$ is the received SINR at the n-th iteration with the transmit power of $P_j^m(n)$, and x is a scalar to adjust the total transmit power. Since (6) has to be satisfied in each iteration as

$$P = \sum_{k=0}^{M-1} P_j^k(n) = \sum_{k=0}^{M-1} P_j^k(n+1) = \sum_{k=0}^{M-1} \frac{x}{\Gamma_j^k(n)} P_j^k(n), \quad (9)$$

we have $x = P / \sum_k (P_j^k(n)/\Gamma_j^k(n))$. Therefore, the recursive update equation is rewritten as

$$P_j^m(n+1) = \frac{P}{\Gamma_j^m(n) \cdot \sum_{k=0}^{M-1} \frac{P_j^k(n)}{\Gamma_j^k(n)}} P_j^m(n). \quad (10)$$

B. Power Allocation Algorithm #2

The algorithm is based on the well-known water-filling theorem [15]. More precisely, defining the sum of the interference and noise power of the m-th subcarrier at the n-th iteration normalized by the channel frequency response including the effect of the beamforming weight as

$$X_j^m(n) = \frac{\sum_{\substack{i=1 \\ i \neq j}}^{N} P_i^m(n) (\mathbf{w}_i^m)^{\mathrm{H}} \boldsymbol{\lambda}_{ji}^m (\boldsymbol{\lambda}_{ji}^m)^{\mathrm{H}} \mathbf{w}_i^m + \sigma_n^2}{(\mathbf{w}_j^m)^{\mathrm{H}} \boldsymbol{\lambda}_{jj}^m (\boldsymbol{\lambda}_{jj}^m)^{\mathrm{H}} \mathbf{w}_j^m}, \quad (11)$$

the algorithm to update the transmit power is summarized as follows;

1) Initialize $K = 0,\ R(n) = \dfrac{P + \sum_m X_j^m(n)}{M}$.
2) If $R(n) > \max\limits_m X_j^m(n)$ then $\forall m$, $P_j^m(n+1) = R(n) - X_j^m(n)$ and exit, otherwise go to 3
3) Define a set $\mathcal{A} = \{m \mid R(n) \leq X_j^m(n)\}$ and $K = \mathrm{Card}(\mathcal{A})$, where Card denotes the number of elements in the set. Modify $R(n)$ as $R(n) = \dfrac{P + \sum_{m \notin \mathcal{A}} X_j^m(n)}{M - K}$, and if $R(n) > \max\limits_{m \notin \mathcal{A}} X_j^m(n)$ then go to 4, otherwise go to 3.
4) $P_j^m(n+1) = \begin{cases} R(n) - X_j^m(n) & (m \notin \mathcal{A}) \\ 0 & (m \in \mathcal{A}) \end{cases}$

V. Discussion on Beamforming Weight

We focus on the transmit beamforming on a certain subcarrier, so the superscript m is dropped and the transmit power P_j^m is set to be 1 in the sequel for the simplicity. The achievable rate on the subcarrier at the j-th mobile terminal is given by

$$R_j(\mathbf{w}_1, \ldots, \mathbf{w}_N) = \log_2 \left(1 + \frac{|\mathbf{w}_j^{\mathrm{H}} \boldsymbol{\lambda}_{jj}|^2}{\sum_{i \neq j} |\mathbf{w}_i^{\mathrm{H}} \boldsymbol{\lambda}_{ji}|^2 + \sigma_n^2} \right), \quad (12)$$

where the bandwidth is normalized to 1. The beamforming weight based on SLNR in (5) can be rewritten as

$$\mathbf{w}_j^{\mathrm{SLNR}} = \left(\sum_{i=1}^{N} \boldsymbol{\lambda}_{ij} (\boldsymbol{\lambda}_{ij})^{\mathrm{H}} + \sigma_n^2 \mathbf{I}_Q \right)^{-1} \boldsymbol{\lambda}_{jj}. \quad (13)$$

In order to investigate the impact of the small change in the beamforming weight from $\mathbf{w}_j^{SLNR}$ on the achievable rate, we obtain following expressions by differentiating R_k $(k = 1, \ldots, N)$ with respect to $\mathbf{w}_j$ $(j = 1, \ldots, N)$.

- for the case $k = j$:

$$\frac{\partial R_j}{\partial \mathbf{w}_j^{\mathrm{H}}} = \frac{1}{\ln 2} \cdot \frac{\boldsymbol{\lambda}_{jj} \boldsymbol{\lambda}_{jj}^{\mathrm{H}} \mathbf{w}_j}{\sum_l |\mathbf{w}_l^{\mathrm{H}} \boldsymbol{\lambda}_{lj}|^2 + \sigma_n^2}. \quad (14)$$

The gradient vector at the maximum SLNR solution is given by

$$\mathbf{r}_{jj} := \left. \frac{\partial R_j}{\partial \mathbf{w}_j^{\mathrm{H}}} \right|_{\mathbf{w}_l = \mathbf{w}_l^{\mathrm{SLNR}},\ \forall l} = p_j \boldsymbol{\lambda}_{jj}, \quad (15)$$

where p_j is a positive number defined as

$$p_j = \frac{1}{\ln 2} \cdot \frac{\boldsymbol{\lambda}_{jj}^{\mathrm{H}} \left(\sum_{l=1}^{K} \boldsymbol{\lambda}_{lj} \boldsymbol{\lambda}_{lj}^{\mathrm{H}} + \sigma_n^2 \mathbf{I}_Q \right)^{-1} \boldsymbol{\lambda}_{jj}}{\sum_l |(\mathbf{w}_l^{\mathrm{SLNR}})^{\mathrm{H}} \boldsymbol{\lambda}_{lj}|^2 + \sigma_n^2}. \quad (16)$$

- for the case $k \neq j$:

$$\frac{\partial R_k}{\partial \mathbf{w}_j^{\mathrm{H}}} = \frac{-\frac{1}{\ln 2} |\mathbf{w}_k^{\mathrm{H}} \boldsymbol{\lambda}_{kk}|^2 \boldsymbol{\lambda}_{kj} \boldsymbol{\lambda}_{kj}^{\mathrm{H}} \mathbf{w}_j}{\left(\sum_l |\mathbf{w}_l^{\mathrm{H}} \boldsymbol{\lambda}_{lj}|^2 + \sigma_n^2 \right) \cdot \left(\sum_{l \neq k} |\mathbf{w}_l^{\mathrm{H}} \boldsymbol{\lambda}_{lj}|^2 + \sigma_n^2 \right)}. \quad (17)$$

The gradient vector at the maximum SLNR solution is given by

$$\mathbf{r}_{jk} := \left.\frac{\partial R_k}{\partial \mathbf{w}_j^{\mathrm{H}}}\right|_{\mathbf{w}_l=\mathbf{w}_l^{\mathrm{SLNR}},\ \forall l}$$
$$= -q_{jk}\boldsymbol{\lambda}_{kj}^{\mathrm{H}}\left(\sum_l \boldsymbol{\lambda}_{lj}\boldsymbol{\lambda}_{lj}^{\mathrm{H}} + \sigma_n^2\mathbf{I}_Q\right)^{-1}\boldsymbol{\lambda}_{jj}\boldsymbol{\lambda}_{kj}, \quad (18)$$

where q_{jk} is a positive number defined as

$$q_{jk} = \frac{\frac{1}{\ln 2}\cdot|(\mathbf{w}_k^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{kk}|^2}{\left(\sum_l |(\mathbf{w}_l^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{lk}|^2+\sigma_n^2\right)\cdot\left(\sum_{l\neq k}|(\mathbf{w}_l^{\mathrm{SLNR}})^{\mathrm{H}}\boldsymbol{\lambda}_{lk}|^2+\sigma_n^2\right)}. \quad (19)$$

From (15), we can see that the rate of the j-th user R_j increases by changing $\mathbf{w}_j^{\mathrm{SLNR}}$ to the direction of $\boldsymbol{\lambda}_{jj}$. We firstly investigate the impact of the change on other users rate R_k $(k \neq j)$ by inner product between $\mathbf{r}_{jk}$ and $\boldsymbol{\lambda}_{jj}$. The inner product is given by

$$(\boldsymbol{\lambda}_{jj})^{\mathrm{H}}\mathbf{r}_{jk} = -q_{jk}\boldsymbol{\lambda}_{kj}^{\mathrm{H}}\left(\sum_l \boldsymbol{\lambda}_{lj}\boldsymbol{\lambda}_{lj}^{\mathrm{H}} + \sigma_n^2\mathbf{I}_Q\right)^{-1}\boldsymbol{\lambda}_{jj}\boldsymbol{\lambda}_{jj}^{\mathrm{H}}\boldsymbol{\lambda}_{kj}, \quad (20)$$

and is less than or equal to 0, since $(\sum_l \boldsymbol{\lambda}_{lj}\boldsymbol{\lambda}_{lj}^{\mathrm{H}} + \sigma_n^2\mathbf{I}_Q)^{-1}$ is a positive definite matrix and $\boldsymbol{\lambda}_{jj}\boldsymbol{\lambda}_{jj}^{\mathrm{H}}$ is a positive semidefinite matrix [16]. The equality holds only when $\boldsymbol{\lambda}_{kj}$ and $\boldsymbol{\lambda}_{jj}$ are orthogonal, while they are random vectors, therefore, we may say the inner product is almost always negative in practice. Therefore, the change of $\mathbf{w}_j^{\mathrm{SLNR}}$ to the direction of $\boldsymbol{\lambda}_{jj}$ in order to increase the rate of the j-th user R_j, almost always results in the reduction of other users rate R_k.

Then, for a general case, we evaluate the impact of changing $\mathbf{w}_j^{\mathrm{SLNR}}$ to the direction of $r\boldsymbol{\lambda}_{jj} + s^*\mathbf{u}_j$, $(r > 0, s \in \mathbb{C})$, where $\mathbf{u}_j$ is an arbitrary vector with $\boldsymbol{\lambda}_{jj}^{\mathrm{H}}\mathbf{u}_j = 0$. The change of the weight increases R_j, because $(r\boldsymbol{\lambda}_{jj} + s^*\mathbf{u}_j)^{\mathrm{H}}\mathbf{r}_{jj} = rp_j||\boldsymbol{\lambda}_{jj}||^2 > 0$. In order to evaluate the sign of inner product

$$(r\boldsymbol{\lambda}_{jj} + s^*\mathbf{u}_j)^{\mathrm{H}}\,\mathbf{r}_{jk}$$
$$= -q_{jk}\boldsymbol{\lambda}_{kj}^{\mathrm{H}}\left(\sum_l \boldsymbol{\lambda}_{lj}\boldsymbol{\lambda}_{lj}^{\mathrm{H}} + \sigma_n^2\mathbf{I}\right)^{-1}\boldsymbol{\lambda}_{jj}(r\boldsymbol{\lambda}_{jj} + s^*\mathbf{u}_j)^{\mathrm{H}}\boldsymbol{\lambda}_{kj}, \quad (21)$$

we consider following two special cases.

Two users ($N = 2$) and two antenna elements ($Q = 2$):

Defining $\boldsymbol{\lambda}_{jj} = [a\ b]^T (a, b \in \mathbb{C})$, we obtain $\mathbf{u}_j = [1/a^*\ \ -1/b^*]^T$ from $\boldsymbol{\lambda}_{jj}^{\mathrm{H}}\mathbf{u}_j = 0$. Therefore, we have

$$\mathbf{A} := \boldsymbol{\lambda}_{jj}(r\boldsymbol{\lambda}_{jj} + s^*\mathbf{u}_j)^{\mathrm{H}} = \begin{bmatrix} r|a|^2 + s & rab^* - s\frac{a}{b} \\ ra^*b + s\frac{b}{a} & r|b|^2 - s \end{bmatrix}.$$

By solving the characteristic equation

$$|\mathbf{A} - \lambda\mathbf{I}| = \lambda\{\lambda - r(|a|^2 + |b|^2)\} = 0,$$

we can see that $\mathbf{A}$ is a positive semidefinite matrix. Since the eigenvector corresponding to the eigenvalue zero is given by $[\ s\frac{a}{b} - rab^*\ \ r|a|^2 + s\ ]^{\mathrm{T}}$, R_k $(k \neq j)$ decreases for the change of the weight, unless $\boldsymbol{\lambda}_{kj}$ is parallel to $[\ s\frac{a}{b} - rab^*\ \ r|a|^2 + s\ ]^{\mathrm{T}}$.

Three users ($N = 3$) and three antenna elements ($Q = 3$):

We consider a case with $N = 3$ and $Q = 3$. Defining $\boldsymbol{\lambda}_{jj} = [a\ b\ c]^{\mathrm{T}}$ and $\mathbf{u}_j = [\alpha\ \beta\ \gamma]^{\mathrm{T}}$, $(a, b, c, \alpha, \beta, \gamma \in \mathbb{C})$, we have

$$\mathbf{A} := \boldsymbol{\lambda}_{jj}(r\boldsymbol{\lambda}_{jj} + s^*\mathbf{u}_j)^{\mathrm{H}} = \begin{bmatrix} r|a|^2 + s\alpha^*a & rab^* + s\beta^*a & rac^* + s\gamma^*a \\ ra^*b + s\alpha^*b & r|b|^2 + s\beta^*b & rbc^* + s\gamma^*b \\ ra^*c + s\alpha^*c & rb^*c + s\beta^*c & r|c|^2 + s\gamma^*c \end{bmatrix}.$$

The characteristic equation is given by

$$|\mathbf{A} - \lambda'\mathbf{I}| = \lambda'^2[\lambda' - \{r(|a|^2 + |b|^2 + |c|^2) + s(\alpha^*a + \beta^*b + \gamma^*c)\}] = 0,$$

while $a^*\alpha + b^*\beta + c^*\gamma = 0$ from $\boldsymbol{\lambda}_{jj}^{\mathrm{H}}\mathbf{u}_j = 0$, therefore, eigenvalues are obtained as $\lambda' = 0$, $r(|a|^2 + |b|^2 + |c|^2)$, and hence $\mathbf{A}$ is a positive semidefinite matrix. Since the eigenvector corresponding to the nonzero eigenvalue is $\boldsymbol{\lambda}_{jj}$, R_k $(k \neq j)$ decreases for the change of the weight, unless $\boldsymbol{\lambda}_{kj}$ is orthogonal to $\boldsymbol{\lambda}_{jj}$.

It should be noted that the sign of the nonzero eigenvalue of $\mathbf{A}$ is solely determined by the real number r for both cases. This means that, if r is set to be negative, the j-th user can increase the rate of other users at the expense of his own rate. From discussions above, it might be concluded that any small change of any user's weight from the SLNR solution almost always results in the reduction of someone's rate.

VI. Numerical Results

In this section, the performance of the transmit beamforming and power allocation methods shown in Sec. III and IV, and the validity of the analysis in Sec. V are examined via computer simulations.

Figs. 1 and 2 show examples of the achievable rate region for two base stations and mobile terminals with two antenna elements ($N = 2$ and $Q = 2$). In the figures, SLNR denotes the rate point achieved by the beamforming vector of (5), while MRC and ZF stands for the rate point achieved by MRC weight and ZF weight, respectively. Since it has been proved that the Pareto Boundary is achieved by the linear combination of the MRC weight and the ZF weight [12], [13], we have evaluated the achievable rate of the linear combination by changing the ratio of the two weights (the dashed line connecting MRC and ZF). We can see that the SLNR based weight and the linear combination with a certain combination ratio can achieve rate points of Pareto boundary, while achieved rate points are different.

Figs. 3 and 4 show the average received SINR of the transmit beamforming combined with the power allocation methods #1 and #2 with the number of antenna elements of $Q = 2$ and $Q = 4$, respectively. For comparison, the SINR for the case with the equal power allocation is also shown in

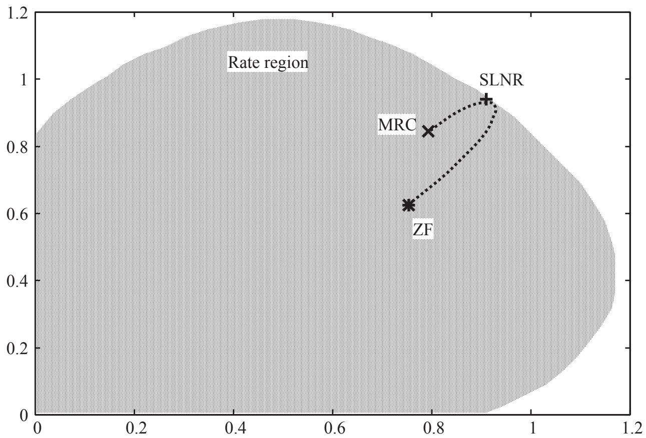

Fig. 1. Example of rate region 1($N = 2$ and $Q = 2$)

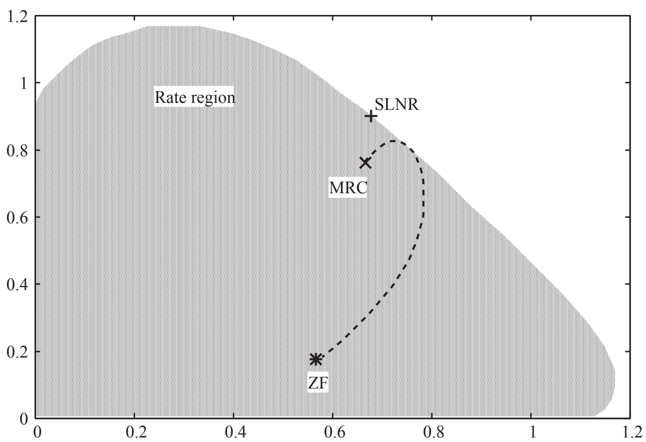

Fig. 2. Example of rate region 2($N = 2$ and $Q = 2$)

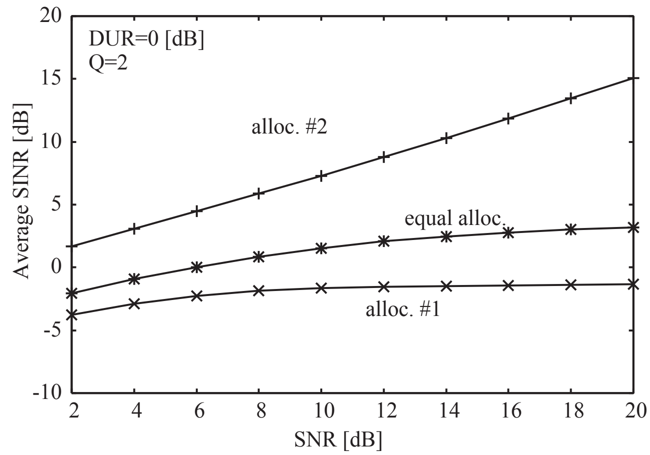

Fig. 3. SINR performance ($P = 2$, DUR= 0dB)

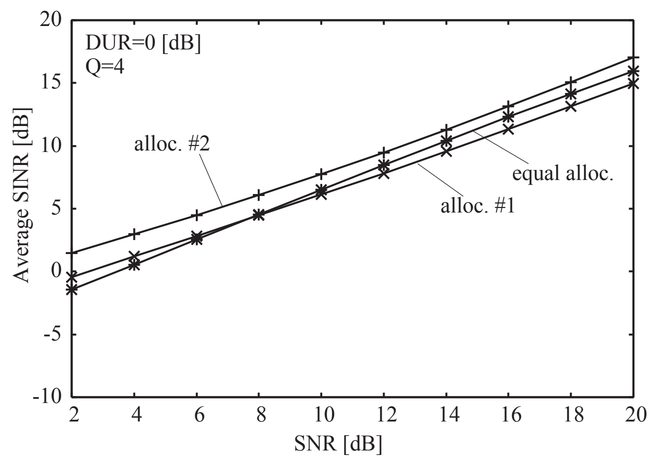

Fig. 4. SINR performance ($P = 4$, DUR= 0dB)

the same figures. In the figures, DUR denotes the power ratio of the desired channel frequency response to the interference channel frequency response $E[|\lambda_{jj}^{m,q}|^2]/E[|\lambda_{ji}^{m,q}|^2]$, and the number of users including the desired user is set to be 3. Note that the number of antenna elements is less than the number of incoming signals for the case of $Q = 2$. We can see that the power allocation method #2 achieves highest SINR among the three methods for both cases, while the average SINR of the allocation method #1 is lower than that of the equal power allocation. This is because the allocation method #1 gives much power to the subcarrier with low SINR in order to improve the BER of the worst subcarriers.

Figs. 5 and 6 show the BER performance of the transmit beamforming with the power allocation method #1 with the number of antenna elements of $Q = 2$ and $Q = 4$, respectively. Here, QPSK with coherent detection is utilized for the Mod. / Demod. scheme. Although the BER is very large for the case of $Q = 2$ with DUR=0 dB for both cases, the power allocation method #1 can considerably improve the BER performance for the rest of cases.

Then, we have compared the sum-rate performance of the transmit beamforming with the two power allocation methods. Here, the sum-rate is defined as

$$\frac{1}{M}\sum_{j=1}^{N}\sum_{m=0}^{M-1}\log_2(1+\hat{\Gamma}_j^m), \tag{22}$$

where $\hat{\Gamma}_j^m$ is the observed SINR of the j-th user on the m-th subcarrier in the simulation. Figs. 7 and 8 show the sum-rate performance with the number of antenna elements of $Q = 2$ and $Q = 4$, respectively. From the figures, we can see that the allocation method #2 can achieve higher sum-rate than the allocation scheme #1 especially when the number of antenna elements is less than the number of incoming signals ($Q = 2$), while the difference in the sum-rate performance between the two allocation methods is rather small for the case of $Q = 4$. It could be attractive to employ the allocation method #1 for the system with enough antenna elements, since the method does not require subcarrier (or subchannel) wise adaptive modulation and coding.

Finally, Figs. 9–12 show typical examples of the achieved power allocation of each user by the methods #1 and #2 for

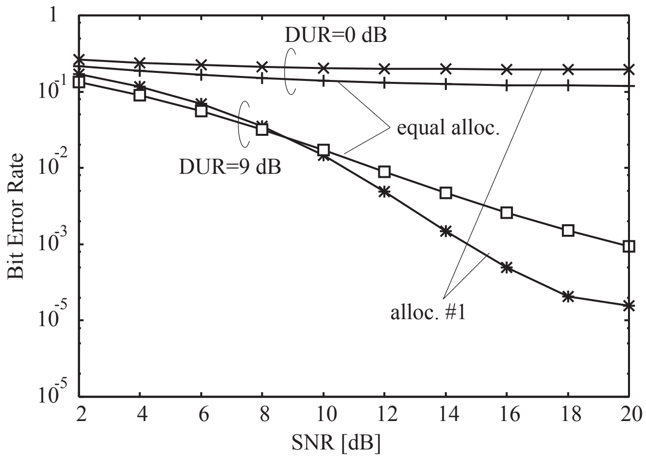

Fig. 5. BER performancepower allocation $\#1$ ($Q = 2$)

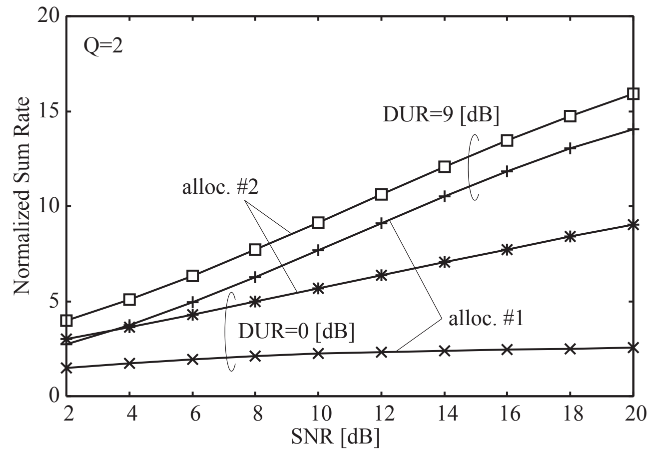

Fig. 7. Achievable sumrate ($Q = 2$)

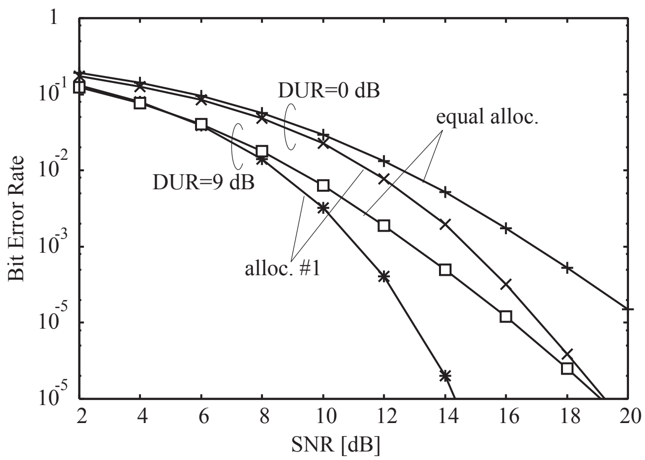

Fig. 6. BER performancepower allocation $\#1$ ($Q = 4$)

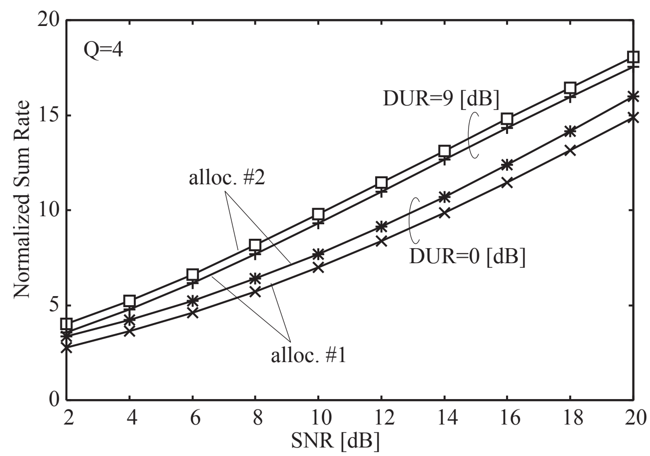

Fig. 8. Achievable sumrate ($Q = 4$)

the cases with the number of antenna elements of $Q = 2$ and $Q = 4$. The SNR and the DUR are set to be 2 dB and 0 dB, respectively. The allocation method $\#1$ tends to allocate larger power on subcarriers, where the other users transmit large power. This is because such subcarriers have large interference power, and the allocation method $\#1$ gives large power to subcarriers with low SINR. On the other hand, the allocation method $\#2$ does not have such tendency, and it decreases the transmit power, if the other users have large power on the subcarrier. Since the transmit power of low SINR subcarrier is set to be 0 by the distributed iterative water-filling algorithm, it can achieve not only the power allocation but also the automatic segregation or subcarrier allocation among the three users. It is interesting to note that the sum-rate performance of the two allocation methods with $Q = 4$ is almost the same as in Fig. 8, while the two allocation methods result in completely different power allocations.

VII. Conclusion

We have considered transmit beamforming and power allocation method for downlink OFDMA systems. The beamforming weight is determined based on the maximization of the SLNR, while the transmit power is allocated by the iterative algorithms based on the equalization of SINRs of subcarriers or the iterative water-filling. We have also discussed the local optimality of the SLNR based beamforming vector. Computer simulation results show that the SLNR based beamforming vector and the linear combination of the MRC and ZF vectors with a certain combination ratio achieve different Pareto optimal points. Moreover, the power allocation method $\#2$ has better performance than the method $\#1$ in terms of the average SINR and the achievable sum-rate, however, the difference is quite small for the case with a sufficient number of antenna elements. Furthermore, the power allocation method $\#2$ can achieve automatic subcarrier segregation among the users for the case of insufficient number of antenna elements.

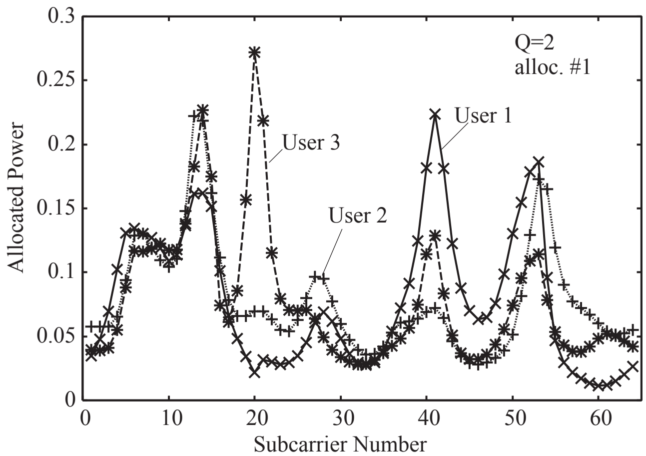

Fig. 9. Example of power allocationpower allocation $\#1$ ($Q = 2$)

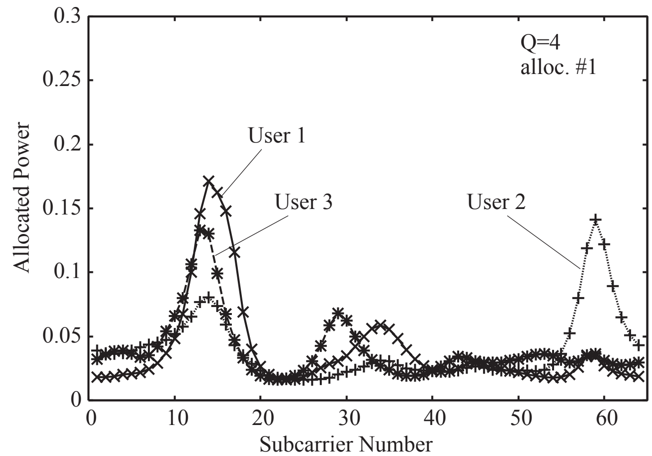

Fig. 11. Example of power allocationpower allocation $\#1$ ($Q = 4$)

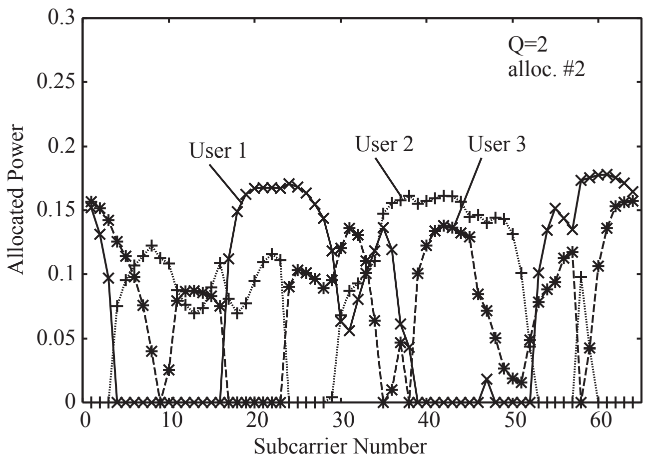

Fig. 10. Example of power allocationpower allocation $\#2$ ($Q = 2$)

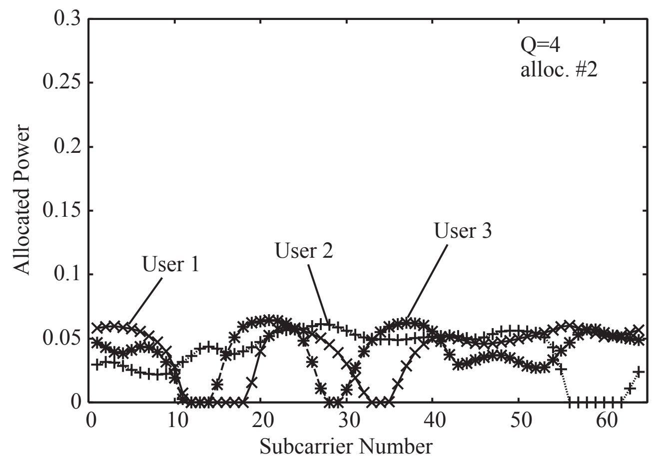

Fig. 12. Example of power allocationpower allocation $\#2$ ($Q = 4$)

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