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SINGLE CARRIER BLOCK TRANSMISSION WITHOUT GUARD INTERVAL

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ABSTRACT

This paper proposes a simple detection scheme for a single carrier block transmission (SCBT) without guard interval (GI). Although the SCBT without the GI can significantly increase bandwidth efficiency, the received signal suffers from inter-block interference (IBI) and inter-symbol interference (ISI), which cannot be effectively equalized by a frequency domain equalizer (FDE). In the proposed scheme, the IBI is cancelled by using a previously detected signal. Moreover, taking advantage of the temporal localization of the difference of the ISI component between the received signal with and without CP, we change the channel matrix from Toeplitz structure into a circulant matrix in order to utilize the conventional FDE. Computer simulation results show that the proposed scheme can outperform a linear MMSE equalizer with lower computational complexity.

I. INTRODUCTION

A block transmission scheme using cyclic prefix (CP) as a guard interval (GI), such as orthogonal frequency division multiplexing (OFDM)[1], digital multitone (DMT)[2] and single carrier block transmission with CP (SC-CP)[3]-[5], has been drawing much attention due to the high performance in frequency selective fading channels with a simple receiver structure using frequency domain equalizer (FDE). However, the CP systems have obtained both of the good performance and the simplicity at the expense of the bandwidth efficiency. If no GI is employed in the block transmission, the received signal suffers from inter-block interference (IBI), and the FDE cannot effectively equalize inter-symbol interference (ISI) because the channel matrix is no more a circulant matrix, although we can expect significant increase in the bandwidth efficiency. So far, some investigations on the CP free block transmission have been made, however, they are mainly for multicarrier systems and employ complex structure of the receiver, such as temporal domain equalizer.

In this paper, we propose a simple detection scheme for the SCBT system without the GI. A block-by-block detection like the SC-CP system is employed in the proposed scheme, therefore, we can utilize a previously detected signal block for the IBI cancellation. Moreover, taking advantage of the temporal localization of the difference of the ISI component between the received signal with and without CP[8], we generate a replica of the difference signal, which enable us to use a conventional FDE by changing the channel matrix from Toeplitz structure into the circulant matrix. Computer simulation results show that the proposed scheme can outperform a linear minimum mean-square-error (MMSE) equalizer while the proposed scheme requires lower computational complexity.

II. SIGNAL MODELING

Let denote the nth transmitted information signal block of size $M \times 1$ as $\mathbf{s}(n) = [s_0(n), \ldots, s_{M-1}(n)]^T$. We do not employ any GI between the transmitted signal blocks. Denoting a channel impulse response as $\{h_0, \ldots, h_L\}$, the nth received signal block $\mathbf{r}(n)$ is given by

$$\mathbf{r}(n) = \mathbf{H}_0\mathbf{s}(n) + \mathbf{H}_1\mathbf{s}(n-1) + \mathbf{n}(n), \tag{1}$$

where $\mathbf{n}(n)$ is a channel noise vector, $\mathbf{H}_0$ and $\mathbf{H}_1$ denote $M \times M$ channel matrices defined as

$$\mathbf{H}_0 = \begin{bmatrix} h_0 & 0 & \ldots & \ldots & \ldots & 0 \\ \vdots & \ddots & \ddots & & & \vdots \\ h_L & & \ddots & \ddots & & \vdots \\ 0 & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & 0 \\ 0 & \ldots & 0 & h_L & \ldots & h_0 \end{bmatrix}, \tag{2}$$

$$\mathbf{H}_1 = \begin{bmatrix} 0 & \ldots & 0 & h_L & \ldots & h_1 \\ \vdots & & & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & h_L \\ \vdots & & & & & 0 \\ \vdots & & & & & \vdots \\ 0 & \ldots & \ldots & \ldots & \ldots & 0 \end{bmatrix}. \tag{3}$$

If we have the GI with sufficient length, $\mathbf{H}_0$ becomes a circulant matrix and $\mathbf{H}_1$ becomes a zero matrix. Therefore, no IBI remains in the received signal block and the FDE can equalize the ISI effectively. However, in our scenario, $\mathbf{H}_0$ and $\mathbf{H}_1$ are no longer a circulant matrix and a zero matrix, respectively.

Defining matrices $\mathbf{C}$, $\mathbf{C}_{ISI}$ and $\mathbf{C}_{IBI}$ as

$$\mathbf{C} = \begin{bmatrix} h_0 & 0 & \ldots & 0 & h_L & \ldots & h_1 \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & h_L \\ h_L & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \ldots & 0 & h_L & \ldots & \ldots & h_0 \end{bmatrix}, \tag{4}$$

$$\mathbf{C}_{ISI} = \mathbf{H}_1, \tag{5}$$

$$\mathbf{C}_{IBI} = \mathbf{H}_1, \tag{6}$$

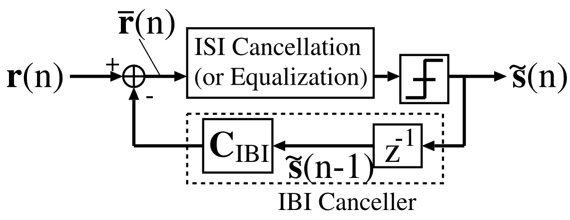

Figure 1: IBI cancellation

we can rewrite the received signal block $\mathbf{r}(n)$ as

$$\mathbf{r}(n) = \mathbf{C}\mathbf{s}(n) - \mathbf{C}_{ISI}\mathbf{s}(n) + \mathbf{C}_{IBI}\mathbf{s}(n-1) + \mathbf{n}(n). \quad (7)$$

The third term in the right hand of (7) is the IBI component in the received signal. Also, since the received signal of the SCBT system with the CP consists of only the first and the fourth term, the second term is a difference ISI component between the received signal with and without the CP.

III. Proposed IBI Cancellation

Since the equalization and the detection are conducted in a block-by-block manner in the proposed scheme, the IBI component $\mathbf{C}_{IBI}\mathbf{s}(n-1)$ could be cancelled by using the previously detected data vector $\tilde{\mathbf{s}}(n-1)$. In the proposed method, we cancel the IBI by subtracting $\mathbf{C}_{IBI}\tilde{\mathbf{s}}(n-1)$ from $\mathbf{r}(n)$ as shown in Fig.1. After the IBI cancellation, the received signal vector $\bar{\mathbf{r}}(n)$ can be written as

$$\begin{aligned} \bar{\mathbf{r}}(n) &= \mathbf{r}(n) - \mathbf{C}_{IBI}\tilde{\mathbf{s}}(n-1), & (8)\\ &\approx (\mathbf{C} - \mathbf{C}_{ISI})\mathbf{s}(n) + \mathbf{n}(n), & (9) \end{aligned}$$

where $\approx$ becomes an equality when $\tilde{\mathbf{s}}(n-1) = \mathbf{s}(n-1)$.

IV. Proposed ISI Cancellation

In this section, we show proposed ISI cancellation (or equalization) methods assuming that the IBI components are completely cancelled, namely,

$$\begin{aligned} \bar{\mathbf{r}}(n) &= (\mathbf{C} - \mathbf{C}_{ISI})\mathbf{s}(n) + \mathbf{n}(n), & (10)\\ &= \mathbf{H}_0\mathbf{s}(n) + \mathbf{n}(n). & (11) \end{aligned}$$

In the followings, we firstly derive a linear MMSE equalizer, which will be a performance benchmark of the other methods. And then, we derive FDE weights for the SCBT system without the GI. Finally, we describe the details of the proposed difference signal cancellation method.

A. Linear Equalization

Denoting the output of the equalizer as $\hat{\mathbf{s}}(n)$, the linear MMSE equalizer can be obtained by minimizing $E\{tr[(\hat{\mathbf{s}}(n) - \mathbf{s}(n))(\hat{\mathbf{s}}(n) - \mathbf{s}(n))^H]\}$, where $E\{\cdot\}$ and $tr[\cdot]$ denote ensemble average and trace of the matrix, respectively. By solving the

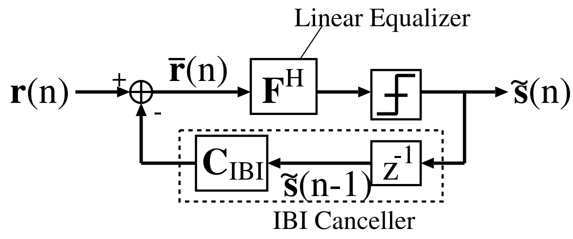

Figure 2: Linear Equalizer

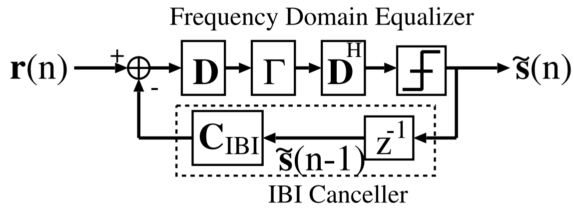

Figure 3: FDE

minimization problem, the linear MMSE equalizer $\mathbf{F}^H$ can be given by

$$\mathbf{F}^H = \mathbf{H}_0^H \cdot \left\{ \mathbf{H}_0\mathbf{H}_0^H + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M \right\}^{-1}, \quad (12)$$

where σ_n^2 and σ_s^2 denote the variances of the noise and the transmitted signal, respectively, and $\mathbf{I}_M$ is an identity matrix of size $M \times M$. Note that the idea itself of the linear equalizer is quite simple, however, it requires high computational complexity due to the inverse of the large ($M \times M$) matrix.

B. 1-tap FDE

The channel matrix $\mathbf{H}_0$ is no more a circulant, therefore, the 1-tap FDE cannot perfectly equalize the distorted received signal even when the IBI is completely cancelled. However, the FDE is still attractive because of the simplicity of the implementation using fast Fourier transform (FFT). Fig.3 shows the configuration of the receiver using the FDE. The output of the FDE can be given by

$$\hat{\mathbf{s}}_{fde}(n) = \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\bar{\mathbf{r}}(n), \quad (13)$$

where $\mathbf{\Gamma} = \text{diag}[\gamma_0, \ldots, \gamma_{M-1}]$ is a diagonal matrix and γ_m is given by (see Appendix)

$$\gamma_m = \frac{\lambda_m^* - g_{m,m}^*}{|\lambda_m|^2 - \lambda_m g_{m,m}^* - \lambda_m^* g_{m,m} + \sum_{i=0}^{M-1} |g_{m,i}|^2 + \frac{\sigma_n^2}{\sigma_s^2}}, \quad (14)$$

$$g_{m,m} = \frac{1}{M}\sum_{l=0}^{L-1}\sum_{i=0}^{l} h_{L-i} e^{j\frac{2\pi}{M}m(M-L+l-i)}, \quad (15)$$

$$\sum_{m=0}^{M-1} |g_{m,n}|^2 = \frac{1}{M}\sum_{l=0}^{L-1}\sum_{i=0}^{l}\sum_{l'=0}^{L-1} |h_{L-i}|^2 e^{j\frac{2\pi}{M}n(l-l')}. \quad (16)$$

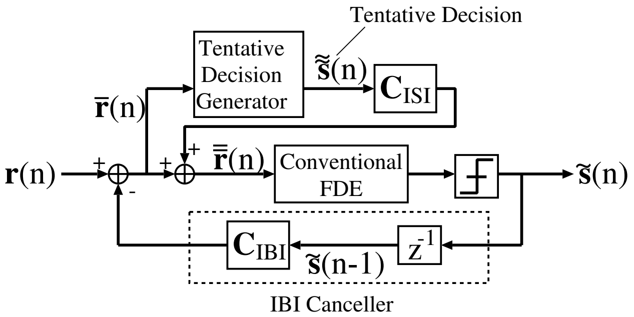

Figure 4: FDE With Difference Signal Cancellation

C. FDE With Difference Signal Cancellation

The proposed FDE requires low computational complexity and can achieve better performance than the conventional FDE, however, it still suffer from performance degradation due to the none-circulant channel matrix $\mathbf{H}_0$. In order to achieve further improvement of the performance, we consider the difference signal $\mathbf{C}_{ISI}\mathbf{s}(n)$ cancellation using tentative decision $\tilde{\tilde{\mathbf{s}}}(n) = [\tilde{\tilde{s}}_0(n), \ldots, \tilde{\tilde{s}}_{M-1}(n)]^T$ as shown in Fig.4. The main idea is that, by adding the replica of $\mathbf{C}_{ISI}\mathbf{s}(n)$ to $\bar{\mathbf{r}}(n)$, we obtain a received signal vector $\bar{\bar{\mathbf{r}}}(n)$, which is distorted only by the circulant matrix $\mathbf{C}$ in the ideal case.

$$\bar{\bar{\mathbf{r}}}(n) = \bar{\mathbf{r}}(n) + \mathbf{C}_{ISI}\tilde{\tilde{\mathbf{s}}}(n), \quad (17)$$
$$\approx \mathbf{C}\mathbf{s}(n) + \mathbf{n}(n). \quad (18)$$

And then, the conventional FDE can efficiently equalize $\bar{\bar{\mathbf{r}}}(n)$ as

$$\hat{\mathbf{s}}_{cancel}(n) = \mathbf{D}^H \mathbf{\Gamma}_{cnv} \mathbf{D} \bar{\bar{\mathbf{r}}}(n), \quad (19)$$

where $\mathbf{\Gamma}_{cnv} = \text{diag}[\gamma_0^{cnv}, \ldots, \gamma_{M-1}^{cnv}]$ is a diagonal matrix of the 1-tap FDE. If we employ the conventional MMSE FDE, the mth element of the equalizer is given by

$$\gamma_m^{cnv} = \frac{\lambda_m^*}{|\lambda_m|^2 + \frac{\sigma_n^2}{\sigma_s^2}}, \quad (20)$$

where λ_m is the mth diagonal element of $\mathbf{\Lambda} = \mathbf{DCD}^H$.

As for the tentative decision used for the replica signal generation, we utilize the structure of $\mathbf{C}_{ISI}$. Since $\mathbf{C}_{ISI}$ has nonzero elements only in L columns,

$$\mathbf{C}_{ISI}\mathbf{s}(n) = \mathbf{C}_{ISI}\begin{bmatrix}\mathbf{0}_{(M-L)\times 1}\\ \mathbf{s}^{sub}(n)\end{bmatrix} \quad (21)$$
$$= \mathbf{C}_{ISI}\mathbf{F}_s\mathbf{F}_s^T\mathbf{s}(n), \quad (22)$$

where $\mathbf{s}^{sub}(n) = [s_{M-L}(n), \ldots, s_{M-1}(n)]^T = \mathbf{F}_s^T\mathbf{s}(n)$ and

$$\mathbf{F}_s = \begin{bmatrix}\mathbf{0}_{(M-L)\times L}\\ \mathbf{I}_L\end{bmatrix}. \quad (23)$$

Therefore, only the corresponding tentative decision $\tilde{\tilde{\mathbf{s}}}^{sub}(n)$, which is defined in the same way as $\mathbf{s}^{sub}(n)$, is required in order to generate the replica of $\mathbf{C}_{ISI}\mathbf{s}(n)$.

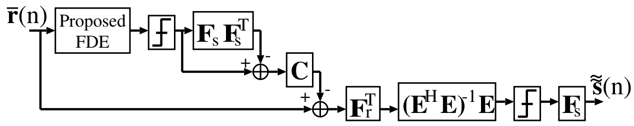

Figure 5: Tentative Decision Generation

Moreover, since

$$\begin{aligned}&\mathbf{H}_0\mathbf{s}(n) - \mathbf{C}\left(\mathbf{I}_M - \mathbf{F}_s\mathbf{F}_s^H\right)\mathbf{s}(n)\\ &= \mathbf{H}_0\mathbf{s}(n) - \mathbf{C}\left(\mathbf{s}(n) - \begin{bmatrix}\mathbf{0}_{(M-L)\times 1}\\ \mathbf{s}^{sub}(n)\end{bmatrix}\right)\\ &= \mathbf{H}_0\begin{bmatrix}\mathbf{0}_{(M-L)\times 1}\\ \mathbf{s}^{sub}(n)\end{bmatrix},\end{aligned} \quad (24)$$

we have

$$\begin{aligned}\bar{\mathbf{r}}'(n) &\stackrel{\text{def}}{=} \bar{\mathbf{r}}(n) - \mathbf{C}\left(\tilde{\mathbf{s}}_{fde}(n) - \begin{bmatrix}\mathbf{0}_{(M-L)\times 1}\\ \tilde{\mathbf{s}}_{fde}^{sub}(n)\end{bmatrix}\right)\\ &\approx \mathbf{H}_0\begin{bmatrix}\mathbf{0}_{(M-L)\times 1}\\ \mathbf{s}^{sub}(n)\end{bmatrix},\end{aligned} \quad (25)$$

where $\tilde{\mathbf{s}}_{fde}(n) = \langle\hat{\mathbf{s}}_{fde}(n)\rangle$ and $\tilde{\mathbf{s}}_{fde}^{sub}(n) = \mathbf{F}_s^T\tilde{\mathbf{s}}_{fde}(n)$. Here, $\langle\cdot\rangle$ denotes detection operation. Furthermore, defining

$$\mathbf{F}_r = \begin{bmatrix}\mathbf{0}_{(M-L)\times L}\\ \mathbf{I}_L\end{bmatrix}, \quad (26)$$

and $\bar{\mathbf{r}}'^{sub}(n) = \mathbf{F}_r^T\bar{\mathbf{r}}'(n)$, we finally have

$$\bar{\mathbf{r}}'^{sub}(n) \approx \mathbf{E}\mathbf{s}^{sub}(n), \quad (27)$$

where

$$\mathbf{E} = \mathbf{F}_r^T\mathbf{H}_0\mathbf{F}_s = \begin{bmatrix}h_0 & & \mathbf{0}\\ \vdots & \ddots & \\ h_{L-1} & \ldots & h_0\end{bmatrix}, \quad (28)$$

By solving (27), the tentative decision for the replica generation can be given by

$$\tilde{\tilde{\mathbf{s}}}^{sub}(n) = \langle\mathbf{E}^{-1}\bar{\mathbf{r}}'^{sub}(n)\rangle. \quad (29)$$

The schematic diagram of the tentative decision generation is shown in Figure 5.

V. Computer Simulation

In order to confirm the performance of the proposed methods, computer simulations are conducted with the following system parameters; Mod./Demod. scheme: QPSK, symbols per block: $M = 64$, channel order: $L = 4$, channel model: 3-path frequency selective Rayleigh fading channel. As for the systems to be evaluated, we consider following 4 systems: the conventional MMSE FDE, the linear MMSE equalizer with the proposed IBI cancellation, the proposed FDE with the IBI cancellation, and the FDE with the IBI and the difference signal

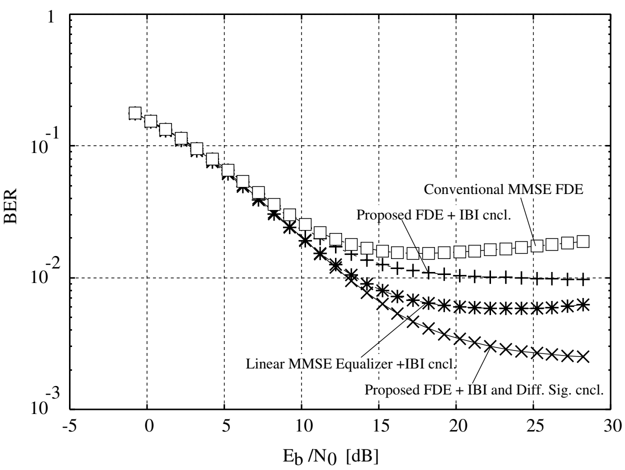

Figure 6: BER Performance

cancellation. Fig.6 shows the BER performances versus the ratio of the energy per bit to the noise power density (E_b/N_0) of the above 4 systems.

From the Figure, we can see that "the FDE with the IBI and the difference signal cancellation" scheme can achieve the best performance among the 4 systems, and amazingly enough, it can outperform "the linear MMSE equalizer with the proposed IBI cancellation". This could be explained by the existence of a nonlinear processing in the proposed difference signal canceller, namely, the detection operation. The nonlinear operation makes it possible for the proposed system to outperform the optimum MMSE linear equalizer. The reason why the BER of the conventional MMSE FDE gets worth as E_b/N_0 increases is that the actual signal to interference plus noise power ratio (SINR) comes away from σ_s^2/σ_n^2 as E_b/N_0 increases.

VI. Conclusion

We have proposed a simple detection scheme for the SCBT system without the GI. Although the GI free block transmission can significantly increase the bandwidth efficiency, it suffer from the IBI and the ISI, which cannot be equalized with the conventional FDE. In the proposed scheme, the IBI is cancelled by using previously detected data signal. As for the ISI, we have considered three ISI cancellation or equalization schemes, namely, the linear MMSE equalizer, the 1-tap FDE, and the FDE with the difference signal cancellation. Moreover, the BER performances of the proposed schemes are evaluated via computer simulations. From the results, we can see that the proposed FDE with the IBI and the difference signal cancellation can outperform the linear MMSE equalizer with lower computational complexity.

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A MMSE WEIGHTS OF 1-TAP FDE FOR SCBT SYSTEM WITHOUT GI

Since the received signal vector can be rewritten as

$$\bar{\mathbf{r}}(n) = \mathbf{D}^H \mathbf{\Lambda} \mathbf{D}\mathbf{s}(n) - \mathbf{C}_{ISI}\mathbf{s}(n) + \mathbf{n}(n), \tag{30}$$

the FDE output can be given by

$$\begin{aligned}\hat{\mathbf{s}}_{fde}(n) =& \mathbf{D}^H \mathbf{\Gamma}\mathbf{\Lambda}\mathbf{D}\mathbf{s}(n) - \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{ISI}\mathbf{s}(n)\\ &+ \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\mathbf{n}(n).\end{aligned} \tag{31}$$

In order to derive the MMSE weights, we define a cost function J to be minimized as $J = E\left\{tr[(\hat{\mathbf{s}}(n) - \mathbf{s}(n))(\hat{\mathbf{s}}^H(n) - \mathbf{s}^H(n))]\right\}$.

Ignoring the term $tr[\sigma_s^2 \mathbf{I}_M]$, which has no elements of $\mathbf{\Gamma}$, J can be written as

$$\begin{aligned}J =& \sigma_s^2\{tr[\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{\Lambda}^H\mathbf{\Gamma}^H] - tr[\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{D}\mathbf{C}_{ISI}^H\mathbf{D}^H\mathbf{\Gamma}^H]\\ &- tr[\mathbf{\Gamma}\mathbf{\Lambda}] - tr[\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{ISI}\mathbf{D}^H\mathbf{\Lambda}^H\mathbf{\Gamma}^H]\\ &+ tr[\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{ISI}\mathbf{C}_{ISI}^H\mathbf{D}^H\mathbf{\Gamma}^H] + tr[\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{ISI}\mathbf{D}^H]\\ &- tr[\mathbf{\Lambda}^H\mathbf{\Gamma}^H] + tr[\mathbf{D}\mathbf{C}_{ISI}^H\mathbf{D}^H\mathbf{\Gamma}^H]\} + \sigma_n^2 tr[\mathbf{\Gamma}\mathbf{\Gamma}^H].\end{aligned} \tag{32}$$

Moreover, defining a matrix as

$$\mathbf{G} = \mathbf{D}\mathbf{C}_{ISI}\mathbf{D}^H, \tag{33}$$

and the (m, n) element of $\mathbf{G}$ as $g_{m,n}$ $(m, n = 0, \ldots, M-1)$, we have

$$\begin{aligned} J =& \sigma_s^2 \sum_{m=0}^{M-1} (|\lambda_m|^2 |\gamma_m|^2 - |\gamma_m|^2 \lambda_m g_{m,m}^* - \gamma_m \lambda_m \\ & - \lambda_m^* |\gamma_m|^2 g_{m,m} + |\gamma_m|^2 \sum_{i=0}^{M-1} |g_{m,i}|^2 + \gamma_m g_{m,m} \\ & - \lambda_m^* \gamma_m^* + g_{m,m}^* \gamma_m^*) + \sigma_n^2 |\gamma_m|^2. \end{aligned} \tag{34}$$

The differentiation of J with respect to γ_m^* is given by

$$\begin{aligned} \frac{\partial J'}{\partial \gamma_m^*} =& \sigma_s^2 \left\{ |\lambda_m|^2 |\gamma_m - \lambda_m g_{m,m}^* \gamma_m - \lambda_m^* g_{m,m} \gamma_m \right. \\ & \left. + \gamma_m \sum_{i=0}^{M-1} |g_{m,i}|^2 - \lambda_m^* + g_{m,m}^* \right\} + \sigma_n^2 \gamma_m. \end{aligned} \tag{35}$$

By solving $\frac{\partial J'}{\partial \gamma_m^*} = 0$, we obtain the MMSE weight (14) as

$$\gamma_m = \frac{\lambda_m^* - g_{m,m}^*}{|\lambda_m|^2 - \lambda_m g_{m,m}^* - \lambda_m^* g_{m,m} + \sum_{i=0}^{M-1} |g_{m,i}|^2 + \frac{\sigma_n^2}{\sigma_s^2}}, \tag{36}$$

where

$$g_{m,m} = \frac{1}{M} \sum_{l=0}^{L-1} \sum_{i=0}^{l} h_{L-i} e^{j \frac{2\pi}{M} m (M-L+l-i)}, \tag{37}$$

and

$$\sum_{m=0}^{M-1} |g_{m,n}|^2 = \frac{1}{M} \sum_{l=0}^{L-1} \sum_{i=0}^{l} \sum_{l'=0}^{L-1} |h_{L-i}|^2 e^{j \frac{2\pi}{M} n (l-l')}. \tag{38}$$