Amplify–and–Forward Cooperative Diversity Schemes for Multi–Carrier Systems

Megumi Kaneko, Kazunori Hayashi, Petar Popovski, Kazushi Ikeda, Hideaki Sakai, and Ramjee Prasad

***Abstract*— We propose generic relay and subcarrier allocation schemes for Multi–Carrier (MC) system with Amplify–and–Forward (AF) relays. The outage probability bounds are derived analytically for each scheme. Simulation results show that these bounds are very tight and better than the bounds obtained straightforwardly from the analysis in the Single–Carrier (SC) case. This is because in our analysis we reckon with the increased degree of freedom brought by the parallel channels. One of the proposed schemes, the Average Best Relay Selection scheme, is best suited for practical implementation since it approaches the best performance while minimizing the required amount of signaling.**

***Index Terms*— Multi–carrier system, orthogonal frequency division multiplex (OFDM), cooperative diversity, relay system.**

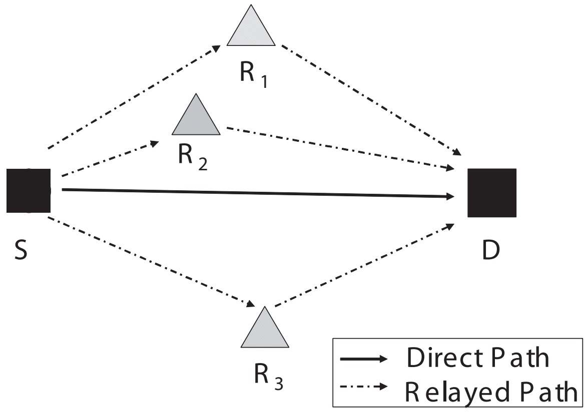

Fig. 1. System model.

I. INTRODUCTION

ONE key challenge for the next generation wireless system is to enable high data rate coverage over large areas. To achieve this, relays are highly envisaged since they can be easily deployed with low–costs [1]. In general, relaying systems are classified as Amplify–and–Forward (AF) or Decode–and–Forward (DF) systems. In AF systems, the relays amplify the received signal without decoding it. In DF systems, the signal is fully decoded and re–encoded prior to retransmission. Relay systems offer a natural setting for cooperative diversity, a form of space diversity exploiting two main features of the wireless medium: its broadcast nature and the spatial independence of its channels [1]. When a source node S sends a message, several relays receive and forward its processed version to the destination node D. D then combines the signals received from the relays, and also the original signal directly received from the source. Many studies have focused on the physical layer issues, such as [2] [3] where distributed space–time codes for a Single–Carrier (SC) system are proposed, or [4] for a Multi–Carrier (MC) system with a single relay.

In [5] a higher layer point of view is taken by proposing two types of relay allocation schemes for the AF system. However, there are no studies about relay allocation using MC transmission, where parallel channel resources are available at the same time. Since MC transmission such as Orthogonal Frequency Division Multiplexing (OFDM) is likely to be a key element in the future wireless communication system, cooperative techniques should be investigated for such a system. Compared to the SC case, the problem at hand is different due to the increase in degrees of freedom brought by OFDM. In this work, different relay allocation schemes using the AF protocol are designed and analyzed. As the amount of channel information increases in a MC system, schemes should be designed while considering the trade–off between the performance and the required amount of information. After introducing the system model, the proposed allocation schemes for MC systems are presented. Next, the theoretical bounding expressions of the system outage are derived. Computer simulations show the validity of this analysis. Finally, the conclusion is drawn and future research directions are suggested.

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II. SYSTEM MODEL

The system considered in this work is depicted in Fig. 1. The source S communicates with the destination D via I relays R_i. Depending on the scheme, there can be 2 to $I+1$ phases. In phase 1, S transmits to D and all R_i. In the next phases, one or more relays transmit to D. S transmits an OFDM symbol of N subcarriers with power P_s. In diversity schemes, the transmitted bits are multiplexed over all the subcarriers which are applied the same modulation level (bit loading schemes are beyond the scope of this paper). Assuming equal power distribution, the received signals at the destination D and relay R_i in the n^{th} subcarrier can be written as

$$\begin{aligned} y_{s,d,n} &= \sqrt{P_{s,n}} h_{s,d,n} x_n + w_{s,d,n} \quad \text{and} \\ y_{s,i,n} &= \sqrt{P_{s,n}} h_{s,i,n} x_n + w_{s,i,n}. \end{aligned} \tag{1}$$

$h_{s,d,n}$ and $h_{s,i,n}$ are the channel fading gains between (S,D) and (S,R_i) on subcarrier n, modelled as a circular symmetric

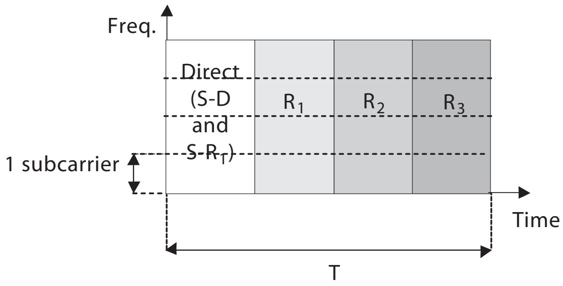

Fig. 2. Frame structure for the APN scheme.

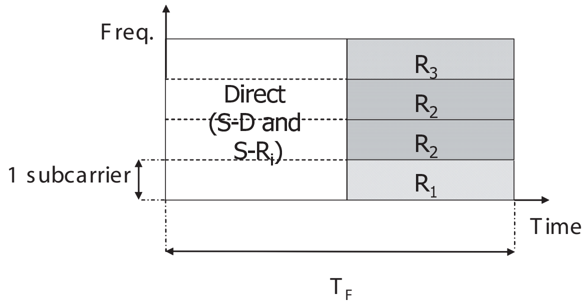

Fig. 3. Frame structure for the NBRS scheme.

complex Gaussian random variable with mean zero and variance one, $w_{s,d,n} \sim CN(0, N_{s,d})$ and $w_{s,i,n} \sim CN(0, N_{s,i})$ are the Additive White Gaussian Noise (AWGN), and x_n denotes the symbol on subcarrier n. In phases 2 to $I+1$, each relay amplifies $y_{s,i,n}$ and transmits it to D with transmission power $P_{i,n}$. Denoting $E[.]$ the operator for ensemble average, the received signal at destination D from relay R_i in subcarrier n is given by [5]

$$y_{i,d,n} = \sqrt{P_{i,n}} h_{i,d,n} \frac{y_{s,i,n}}{\sqrt{E[|y_{s,i}|^2]}} + w_{i,d,n}. \tag{2}$$

Note that, since the relay transmit power is fixed, the amplifying gain depends on the channel fading gain $h_{s,i,n}$ between the source and the relay. $w_{i,d,n} \sim CN(0, N_{i,d})$ denotes the AWGN in the (R_i,D) channel in subcarrier n. By assuming ideal Maximum Ratio Combining (MRC) at the destination, the Signal–to–Noise–Ratio (SNR) at the combiner output is the sum of the instantaneous SNR of all subcarriers and relays. By defining as in [5], $\gamma = 1/N_0$, $N_{s,d} = N_0$, $N_{s,i} = k_{s,i}N_0$, $N_{i,d} = k_{i,d}N_0$, $\alpha_{0,n} = P_{s,n}|h_{s,d,n}|^2$, $\alpha_{i,n} = P_{s,n}|h_{s,i,n}|^2/k_{s,i}$ and $\beta_{i,n} = P_{s,n}|h_{i,d,n}|^2/k_{i,d}$, the SNR of the direct link and the SNR of a relayed link summed over subcarriers can be expressed as $\gamma_0 = \sum_{n=1}^{N} \gamma\alpha_{0,n}$ and $\gamma_i = \sum_{n=1}^{N} \frac{\gamma\alpha_{i,n}\gamma\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}}$, respectively.

III. Proposed Cooperative Diversity Schemes

The first two schemes, *All–Participate All Subcarrier (APN)* and *All–Participate Rate Splitting (AP–RS)*, use the frame depicted in Fig. 2, for $I = 3$. The frame of duration T_F is equally divided between S and all R_i. In phase 1, S transmits to D and to all R_i. In the next phases, the I relays transmit sequentially. S and R_i use all the subcarriers within their allocated frame portion. APN and AP–RS differ in the way of multiplexing the data over the subcarriers. In APN, the data is spread accross all the subcarriers and then sent. Thus, APN is in outage if the total rate resulting from the sum of the SNR of all subcarriers is smaller than the target rate. The APN scheme extends the SC AP–AF scheme in [5]. In AP–RS, the total data is split into N equal streams transmitted from each subcarrier at the same rate, equal to the target rate R divided by N. Since all the subcarriers have to carry a data rate larger than R/N, AP–RS is in outage if any of the subcarriers has a rate smaller than R/N.

The third scheme, *Average Best Relay Selection Scheme (AvgBRS)*, is the extension to the MC case of the S–AF scheme in [5]. The frame is divided in two and the relay with the best SNR averaged over all subcarriers transmits in phase 2.

The fourth scheme is *Per–Subcarrier Best Relay Selection Scheme (NBRS)*, shown in Fig. 3. Transmission occurs also in two phases: S transmits, and then the best relay in each subcarrier transmits. NBRS may not be practical as it requires a higher complexity and the knowledge of the CSI for all relays and subcarriers, which must be sent to S via an uplink feedback channel. However, NBRS gives a better performance compared to APN and AvgBRS since the allocation is adapted per–subcarrier. Subcarrier reordering could be performed at the relay, but this operation increases the processing complexity at the relays, thereby hindering the use of AF protocol.

Finally, the *Random Relay Selection (RRS)* scheme, provides a reference to the performance of the schemes presented above. It is a two–phased transmission, where the source transmits during the first half of the frame, and then a randomly chosen relay transmits in the second half.

IV. Preliminary Analysis

We derive an essential result used in the following outage probability analysis. It is a generalization of the one in [3] for multiple carriers and/or relays: Let $r_\delta(N)=\sum_{n=1}^{N} \delta f(v_n/\delta, w_n/\delta)$ with δ positive, where $f(x,y)=(xy)/(x+y+1)$ and v_n and w_n are independent exponential random variables with parameters λ_n, μ_n $\forall$ n, respectively. Let $h(\delta) > 0$ be continuous with $h(\delta) \to 0$ and $\delta/h(\delta) \to k < \infty$ as $\delta \to 0$. Then the probability $P[r_\delta(N) < h(\delta)]$ satisfies

$$\begin{aligned}
\liminf_{\delta\to 0} \frac{1}{h^N(\delta)} P\left[r_\delta(N) < h(\delta)\right] &\geq \frac{1}{N!}\prod_{n=1}^{N}(\lambda_n+\mu_n) \\
\limsup_{\delta\to 0} \frac{1}{h^N(\delta)} P\left[r_\delta(N) < h(\delta)\right] &\leq \prod_{n=1}^{N}(\lambda_n+\mu_n).
\end{aligned} \tag{3}$$

The proof is outlined in the Appendix. For the SC case, where $r_\delta(1)=\delta f(v_1/\delta, w_1/\delta)$ and λ_1, μ_1 are the parameters of the exponential random variables v_1 and w_1, it was shown in [3] that

$$\lim_{\delta\to 0} \frac{1}{h(\delta)} P[r_\delta(1) < h(\delta)] = \lambda_1 + \mu_1. \tag{4}$$

V. Theoretical Analysis of Outage Probability

A. All–Participate All Subcarrier Scheme (APN)

The capacity achieved by *APN* can be written as

$$C_{APN} = \frac{1}{I+1}\log_2\Big(1+\sum_{n=1}^{N}\gamma\alpha_{0,n} + \sum_{n=1}^{N}\sum_{i=1}^{I}\frac{\gamma\alpha_{i,n}\gamma\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}}\Big), \quad (5)$$

By defining $\tilde{d}$=$\frac{2^{(I+1)R}-1}{\gamma}$, the outage probability, e. g., the probability that the capacity C_{APN} falls below a predetermined rate R, can be expressed as

$$P_{out}^{APN} = P\left[\sum_{n=1}^{N}\alpha_{0,n} + \sum_{i=1}^{I}\sum_{n=1}^{N}\frac{\gamma\alpha_{i,n}\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}} < \tilde{d}\right]. \quad (6)$$

Since we assume Rayleigh fading channels $h_{s,d,n}, h_{s,i,n}, h_{i,d,n} \sim \mathcal{CN}(0,1)$, the weighted amplitude squares $\alpha_{0,n}$, $\alpha_{i,n}$ and $\beta_{i,n}$ are exponentially distributed with parameters $\lambda_{0,n}$=λ_0, $\lambda_{i,n}$ and $\mu_{i,n}$, respectively, defined as $\lambda_0 = \frac{1}{P_{s,n}}$, $\lambda_{i,n} = \frac{k_{s,i}}{P_{s,n}}$ and $\mu_{i,n} = \frac{k_{i,d}}{P_{s,n}}$. As a sum of N independent random exponential variables with parameter λ_0, the probability density function of x=$\sum_{n=1}^{N}\alpha_{0,n}$ is equal to $p_x(x) = \frac{\lambda_0^N}{(N-1)!}e^{-\lambda_0 x}x^{N-1}$ [6].

With a change of variable x'=$1-x/\tilde{d}$, (6) becomes

$$P_{out}^{APN} = \tilde{d}^{(I+1)N}\int_0^1 \frac{P\left[\sum_{i=1}^{I}\sum_{n=1}^{N}\frac{\gamma\alpha_{i,n}\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}} < \tilde{d}x'\right]}{(\tilde{d}x')^{IN}} \times\frac{\lambda_0^N}{(N-1)!}e^{-\lambda_0 x'\tilde{d}(1-x')}x'^{IN}(1-x')^{N-1}dx'. \quad (7)$$

Eq. (3) enables to take the $\liminf$ of the probability over the summations over I and N,

$$\frac{1}{(IN)!}\prod_{i=1}^{I}\prod_{n=1}^{N}(\lambda_{i,n}+\mu_{i,n}) \le \liminf_{\gamma\to\infty}\frac{P\left[\sum_{i=1}^{I}\sum_{n=1}^{N}\frac{\gamma\alpha_{i,n}\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}} < \tilde{d}x'\right]}{(\tilde{d}x')^{IN}}. \quad (8)$$

Since $\lim_{\gamma\to\infty} e^{-\lambda_0 x'\tilde{d}(1-x')} = 1$ for high SNR, the lower bound L_{APN} becomes

$$L_{APN} = \tilde{d}^{(I+1)N}\frac{\lambda_0^N}{(N-1)!}\times\frac{1}{(IN)!}\prod_{i=1}^{I}\prod_{n=1}^{N}(\lambda_{i,n}+\mu_{i,n}) \times\int_0^1 x'^{IN}(1-x')^{N-1}dx'. \quad (9)$$

Since $\int_0^1 x'^{IN}(1-x')^{N-1}dx'$=$\frac{(N-1)!(IN)!}{(IN+N)!}$, the lower bound is obtained, as well as the upper bound in the same way

$$L_{APN} = \frac{\lambda_0^N}{(IN+N)!}\prod_{i=1}^{I}\prod_{n=1}^{N}(\lambda_{i,n}+\mu_{i,n})\,\tilde{d}^{(I+1)N}$$
$$\text{and}\quad U_{APN} = (IN)!\times L_{APN}. \quad (10)$$

Without the result from (3), the direct application of the limit (4) derived in [3] requires the isolation of each term in the summation. In that case, we consider the following inequalities

$$\max_{\substack{i\in[1..I]\\ n\in[1..N]}}\frac{\gamma^2\alpha_{i,n}\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}} \le \sum_{i=1}^{I}\sum_{n=1}^{N}\frac{\gamma^2\alpha_{i,n}\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}} \le IN\times\max_{\substack{i\in[1..I]\\ n\in[1..N]}}\frac{\gamma^2\alpha_{i,n}\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}}. \quad (11)$$

By applying (4), the following bounds are derived,

$$L_{APN}^{Ref} = \frac{\lambda_0^N(IN)!}{(IN+N)!(IN)^{IN}}\prod_{i=1}^{I}\prod_{n=1}^{N}(\lambda_{i,n}+\mu_{i,n})\,\tilde{d}^{(I+1)N}$$
$$\text{and } U_{APN}^{Ref} = L_{APN}^{Ref}\times(IN)^{IN}. \quad (12)$$

While $U_{APN} = U_{APN}^{Ref}$, it is clear that the lower bound L_{APN} is tighter than L_{APN}^{Ref} which will be used as reference, since $L_{APN}^{Ref} = L_{APN}\times\frac{(IN)!}{(IN)^{IN}}$ and $\frac{(IN)!}{(IN)^{IN}} < 1$.

B. All–Participate Rate Splitting Scheme (AP–RS)

As explained in section III, this scheme is in outage if one of the subcarriers carry a rate smaller than the target rate divided by the number of subcarriers. If r_n is the rate carried by subcarrier n, the outage probability can be written $P_{out}^{APRS} = 1 - P\left[\forall n\in[1..N], r_n \ge \frac{R(I+1)}{N}\right]$, since each subcarrier is shared between the all the relays and the source. By assuming the subcarriers independent and defining $\delta = \frac{2^{\frac{R(I+1)}{N}}-1}{\gamma}$, P_{out}^{APRS} becomes

$$P_{out}^{APRS} = 1-\prod_{n=1}^{N}P\left[r_n \ge \frac{R(I+1)}{N}\right] = 1-\prod_{n=1}^{N}P\left[\sum_{i=1}^{I}\frac{\alpha_{i,n}\gamma\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}} \ge \delta-\alpha_{0,n}\right], \quad (13)$$

With the same method as in section V-A, the bounds can be expressed as in (14).

C. Average Best Relay Selection Scheme (AvgBRS)

Posing $d = \frac{2^{2R}-1}{\gamma}$, the capacity and outage probability achieved by *AvgBRS* can be written as

$$C_{AvgBRS} = \frac{1}{2}\log_2\Big(1+\sum_{n=1}^{N}\gamma\alpha_{0,n} + \max_{i\in[1..I]}\sum_{n=1}^{N}\frac{\gamma\alpha_{i,n}\gamma\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}}\Big) \quad (15)$$

$$P_{out}^{AvgBRS} = P\Big[\sum_{n=1}^{N}\alpha_{0,n} + \max_{i\in[1..I]}\sum_{n=1}^{N}\frac{\gamma\alpha_{i,n}\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}} < d\Big] \quad (16)$$

$$L_{APRS} = 1 - \prod_{n=1}^{N}\left[1 - \frac{\lambda_0}{I+1} \times \frac{1}{I!}\prod_{i=1}^{I}(\lambda_{i,n}+\mu_{i,n}) \times \delta^{I+1}\right]$$
$$U_{APRS} = 1 - \prod_{n=1}^{N}\left[1 - \frac{\lambda_0}{I+1}\prod_{i=1}^{I}(\lambda_{i,n}+\mu_{i,n}) \times \delta^{I+1}\right]. \quad (14)$$

As for *APN* scheme, we derive the lower bound for *AvgBRS* scheme using (3),

$$\prod_{i=1}^{I}\left(\frac{1}{N!}\prod_{n=1}^{N}(\lambda_{i,n}+\mu_{i,n})\right) \leq \liminf_{\gamma\to\infty}\prod_{i=1}^{I}\left(\frac{P[\sum_{n=1}^{N}\frac{\gamma\alpha_{i,n}\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}} < dx']}{(dx')^N}\right). \quad (17)$$

This leads to the following lower bound, and the upper bound using the same method (18).

D. Per–Subcarrier Best Relay Selection Scheme (NBRS)

For this scheme, the achieved capacity and outage probability can be written as

$$C_{NBRS} = \frac{1}{2}\log_2\Big(1 + \sum_{n=1}^{N}\gamma\alpha_{0,n} + \sum_{n=1}^{N}\max_{i\in[1..I]}\frac{\gamma\alpha_{i,n}\gamma\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}}\Big), \quad (19)$$

$$P_{out}^{NBRS} = P\Big[\sum_{n=1}^{N}\alpha_{0,n} + \sum_{n=1}^{N}\max_{i\in[1..I]}\frac{\gamma\alpha_{i,n}\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}} < d\Big] \quad (20)$$

Using the same method as previously, we derive the outage bounds using the inequalities below

$$\max_{\substack{i\in[1..I]\\ n\in[1..N]}}\frac{\gamma^2\alpha_{i,n}\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}} \leq \sum_{n=1}^{N}\max_{i\in[1..I]}\frac{\gamma^2\alpha_{i,n}\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}} \leq N \times \max_{\substack{i\in[1..I]\\ n\in[1..N]}}\frac{\gamma^2\alpha_{i,n}\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}} \quad (21)$$

By using the limit derived in [3] and with similar derivations, we obtain

$$L_{BRS}^{Ref} = \frac{\lambda_0^N (IN)!}{(IN+N)!N^{IN}}\prod_{i=1}^{I}\prod_{n=1}^{N}(\lambda_{i,n}+\mu_{i,n})\, d^{(I+1)N}$$
$$\text{and} \quad U_{BRS}^{Ref} = L_{BRS}^{Ref} \times N^{IN}. \quad (22)$$

These bounds will be referred to as *reference bounds* since they are derived from the limit (4) from [3], using a similar approach as in [5]. We can also see from (10), (18) and (22) that the APN, AvgBRS and NBRS schemes all achieve full diversity order of $(I+1)N$.

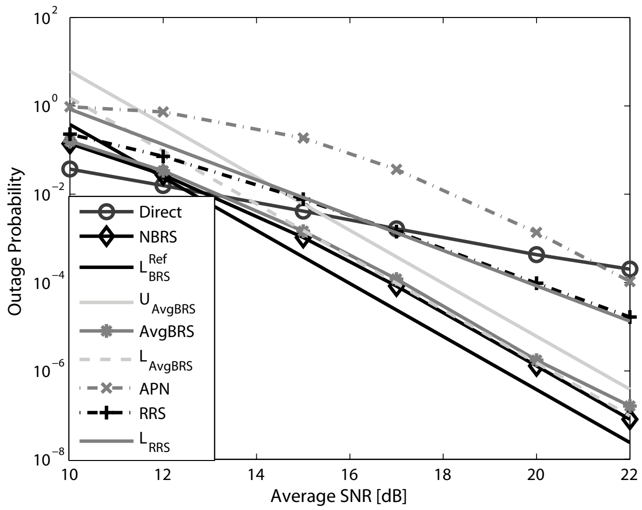

Fig. 4. Outage Probability and bounds for the proposed schemes.

E. Random Relay Selection Scheme (RRS)

Posing $d = \frac{2^{2R}-1}{\gamma}$, the outage probability for the RRS scheme can be written

$$P_{out}^{RRS} = P\left[\sum_{n=1}^{N}\alpha_{0,n} + \underset{i\in[1..I]}{\mathrm{rand}}\sum_{n=1}^{N}\frac{\gamma\alpha_{i,n}\beta_{i,n}}{1+\gamma\alpha_{i,n}+\gamma\beta_{i,n}} < d\right]. \quad (23)$$

The function rand picks a random discrete value in $[1..I]$. The derivations and (3) lead to

$$L_{RRS} = \frac{\lambda_0^N N}{(2N)!}\sum_{i=1}^{I}\frac{1}{I}\frac{1}{N!}\prod_{n=1}^{N}(\lambda_{i,n}+\mu_{i,n})\, d^{2N} \text{ and}$$
$$U_{RRS} = \frac{\lambda_0^N N}{(2N)!}\sum_{i=1}^{I}\frac{1}{I}\prod_{n=1}^{N}(\lambda_{i,n}+\mu_{i,n})\, d^{2N}. \quad (24)$$

These expressions show that RRS scheme achieves a diversity order of $2N$ at most, coming from the N subcarriers of the direct path and of the chosen relay. However, not more than order N diversity can be guaranteed for the relayed path since the relay is chosen randomly.

VI. Numerical Results

The goal here is twofold: first, to compare the performance of the proposed schemes and second, to assess the analytically derived bounds for high SNR.

Simulations are made assuming $I = 2$, $N = 2$, and the target rate R was fixed to 2 bits (1 bit per subcarrier). Rayleigh fading channels with parameters $h_{s,d,n}, h_{s,i,n}, h_{i,d,n} \sim \mathcal{CN}(0,1)\,\forall(i,n)$ are considered, so that the parameters of the exponential random variables $\alpha_{0,n}$, $\alpha_{i,n}$, $\beta_{i,n}$ verify

$$L_{AvgBRS} = \frac{\lambda_0^N (IN)!}{(IN+N)!} \times \frac{1}{(N!)^I} \prod_{i=1}^{I} \prod_{n=1}^{N} (\lambda_{i,n} + \mu_{i,n})\, d^{(I+1)N}$$
$$U_{AvgBRS} = \frac{\lambda_0^N (IN)!}{(IN+N)!} \prod_{i=1}^{I} \prod_{n=1}^{N} (\lambda_{i,n} + \mu_{i,n})\, d^{(I+1)N} \quad (18)$$

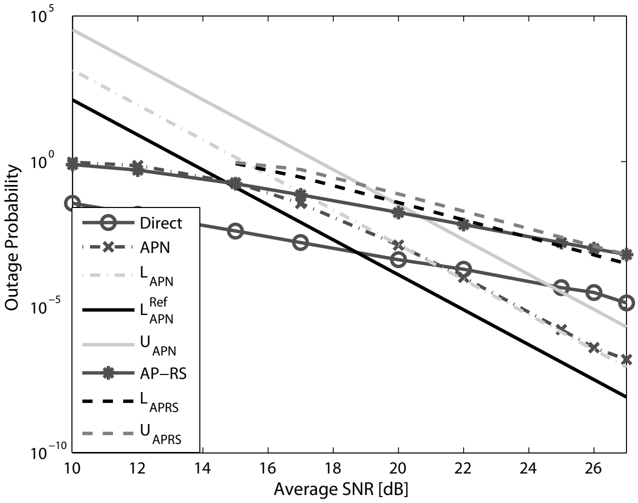

Fig. 5. Outage Probability and bounds for APN and AP–RS schemes.

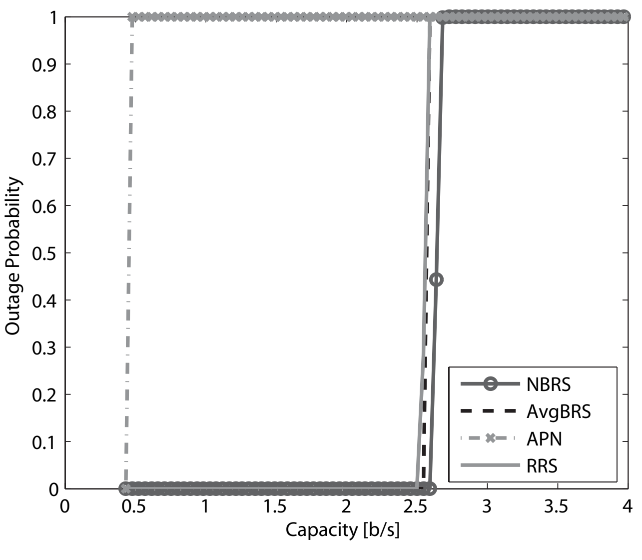

Fig. 6. CDF of the capacity [b/s] of the proposed schemes, OFDM channel $N = 512$, $I = 12$.

λ_0=$\lambda_{i,n}$=$\mu_{i,n}$=1, $\forall(i,n)$. Fig. 4 shows the outage performance of the four schemes APN, AvgBRS, NBRS and RRS, for different values of the system SNR γ, constant and common to all links. We can observe that APN performs very poorly compared to AvgBRS and NBRS, due to the time partition between all access points which degrades the throughput and thus the outage probability. Moreover, the outage of AvgBRS closely approaches that of NBRS. AvgBRS provides a near–optimal performance while drastically decreasing the required amount of CSI: AvgBRS only needs the feedback of the ID of the best relay which requires $\log_2(I)$ bits, whereas NBRS needs the ID of the best relay for each subcarrier, which amounts to $N \times \log_2(I)$ bits.

Compared to RRS, AvgBRS achieves a much better performance with a low amount of CSI, showing that it pays off to spend resources for feedback and chose the best relay in average. Moreover, the simulated curve of AvgBRS is well bounded at high SNR by the analytically derived L_{AvgBRS} and U_{AvgBRS}. In addition, the derived lower bounds improve drastically the reference ones. Comparing (15) and (21), it can be seen that L_{BRS}^{Ref} (22) serves as a reference lower bound for NBRS but also for AvgBRS. Fig. 4 shows that L_{AvgBRS} is much tighter than L_{BRS}^{Ref} and L_{RRS} also closely approaches the outage probability of RRS.

Fig. 5 shows the outage probability of APN and AP–RS schemes. APN performs much better than AP–RS, except for SNR values lower than 15dB, where AP–RS performs slightly better. The simulated curves and the derived bounds (10) show that coding the data accross all the subcarriers provides full spatial and multipath diversity, which is not the case with rate splitting. That is, if one subcarrier supports a rate lower than R/N, the whole scheme is in outage, e.g., there is no diversity gain from the subcarriers, which explains the poor outage performance. The bounds for AP–RS, L_{APRS} and U_{APRS} are not plotted for low SNR since they take negative values (see (14)). At high SNR, the bounds match well the simulated performance of AP–RS. Concerning APN, the derived lower bound L_{APN} is much tighter than the reference bound L_{APN}^{Ref} and provides a very good approximation of the APN outage.

Now, simulations are made with a practical OFDM channel, generated by the multipath Rayleigh fading channel model with exponential power delay profile. The Cumulative Density Function (CDF) of the capacity is evaluated for APN, AvgBRS, NBRS and RRS schemes, with $N = 512$ subcarriers and $I = 12$ relays, for a fixed average SNR level of 15 dB in all links. Fig. 6 shows that AvgBRS approaches the performance of NBRS, and both schemes outperform APN. While the performance of RRS and AvgBRS is comparable, due to the reduced gap in the SNR averaged over subcarriers for different relays for a large N, (18) and (24) show that AvgBRS achieves full diversity order $((N+1)I)$ for only $2N$ for RRS. Assuming a realistic OFDM channel model, AvgBRS achieves the best compromise, having a near–optimal performance with a minimal amount of CSI feedback.

VII. CONCLUSION

Several allocation schemes were proposed using cooperative diversity for the multi–carrier AF relay system. Theoretical bounds at high SNR of the outage probability were derived via analysis. Simulations have shown the validity of the derived bounds, and in particular the derived lower bounds are much tighter than the reference ones. Moreover, the *AvgBRS* scheme had a near–optimal performance as it closely approached the *NBRS* scheme, while requiring N times less CSI feedback. In the future work, adaptive resource allocation methods will be investigated, based on these generic ones, assuming multiple users.

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Appendix: Proof of Theorem 3

Given independent exponential random variables v_n, w_n, with parameters $\lambda_n, \mu_n\ \forall\ n \in [1..N]$,

$$r_\delta(N) = \sum_{n=1}^{N} \frac{1}{\frac{1}{v_n} + \frac{1}{w_n} + \frac{\delta}{v_n w_n}} = \sum_{n=1}^{N} u_n, \qquad (25)$$

and the PDF of u_n is denoted $p_{u_n}(u_n)$. Following the same approach as in [3], for any $x > 0$,

$$\Pr[u_n < x] \geq 1 - \exp[-(\lambda_n + \mu_n)x], \qquad (26)$$

which can be expressed in integral form as

$$\int_0^x p_{u_n}(u_n) du_n \geq \int_0^x (\lambda_n + \mu_n) \exp[-(\lambda_n + \mu_n)u_n] du_n. \qquad (27)$$

By setting $x = \epsilon u$ ($\epsilon > 0, u > 0$) and taking the limit of $\epsilon \to 0$, we obtain

$$\begin{aligned} &\liminf_{\epsilon \to 0} \int_0^{\epsilon u} p_{u_n}(u_n) du_n \\ &\geq \lim_{\epsilon \to 0} \int_0^{\epsilon u} (\lambda_n + \mu_n) \exp[-(\lambda_n + \mu_n)u_n] du_n, \end{aligned} \qquad (28)$$

which implies

$$\begin{aligned} &\liminf_{\epsilon \to 0} p_{u_n}(\epsilon u) \\ &\geq \lim_{\epsilon \to 0} (\lambda_n + \mu_n) \exp[-(\lambda_n + \mu_n)\epsilon u] = \lambda_n + \mu_n. \end{aligned} \qquad (29)$$

Thus, $p_{u_n}(u_n)$ satisfies condition (30) in [2] and from (41) in [2], we have

$$\liminf_{\delta \to 0} \frac{1}{h^N(\delta)} P[r_\delta(N) < h(\delta)] \geq \frac{1}{N!} \prod_{n=1}^{N} (\lambda_n + \mu_n). \qquad (30)$$

For the upper bound, we observe

$$\begin{aligned} r_\delta(N) &= \sum_{n=1}^{N} \frac{1}{\frac{1}{v_n} + \frac{1}{w_n} + \frac{\delta}{v_n w_n}} \\ &\geq \max_n \left(\frac{1}{\frac{1}{v_n} + \frac{1}{w_n} + \frac{\delta}{v_n w_n}} \right), \end{aligned} \qquad (31)$$

which implies

$$P[r_\delta(N) < h(\delta)] \leq \prod_{n=1}^{N} P\left[\frac{1}{v_n} + \frac{1}{w_n} + \frac{\delta}{v_n w_n} > \frac{1}{h(\delta)}\right] \qquad (32)$$

From [3], we know that

$$\frac{1}{h(\delta)} P\left[\frac{1}{v_n} + \frac{1}{w_n} + \frac{\delta}{v_n w_n} > \frac{1}{h(\delta)}\right] \leq \lambda_n + \mu_n,\ \forall\ n. \qquad (33)$$

As $\frac{1}{h^N(\delta)} P[r_\delta(N) < h(\delta)]$ is bounded and sup is a monotonically non–increasing function, we finally obtain

$$\limsup_{\delta \to 0} \frac{1}{h^N(\delta)} P[r_\delta(N) < h(\delta)] \leq \prod_{n=1}^{N} (\lambda_n + \mu_n). \qquad (34)$$

Note that in [2], u_n is mainly assumed as an exponential random variable, whereas here we show (30) for a far more complex u_n. That is why the lower and upper bounds are not equal.

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