Downlink Channel Estimation for Multi-cell Block Transmission Systems with Cyclic Prefix

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***Abstract*— This paper investigates downlink channel estimation schemes for multi-cell block transmission systems with cyclic prefix and proposes a time domain channel estimation scheme using iterative least square interference cancellation. The proposed scheme takes advantage of the fact that each received pilot signal exists in up to the length of the cyclic prefix pulse one dimensional subspace in the fast Fourier transform (FFT) size dimensional received signal vector space. Computer simulation results show that the proposed scheme can achieve almost the same channel estimation accuracy in multi-cell scenario as that in single-cell environment with small number of iterations.**

I. Introduction

Block transmission schemes with cyclic prefix, such as orthogonal frequency division multiplexing (OFDM)[1] and single carrier block transmission with cyclic prefix (SC-CP)[2]-[4], have been drawing much attention because of the effective and simple frequency domain equalizer (FDE) using fast Fourier transform (FFT). Of particular significance of the block transmission systems with the cyclic prefix is that, as far as the length of the guard interval (GI) is greater than or equal to the channel order, the channel matrix is given by a circulant matrix, which is diagonalized by a discrete Fourier transform (DFT) matrix.

Recently, much effort to apply the block transmission schemes with cyclic prefix to mobile communications systems has been made as typified by WiMAX system, where IEEE 802.16e standard[5] is adopted for the physical layer/medium access control layer protocol, and by fourth generation mobile communications system. One of the most serious problems in such mobile communications environment is the existence of interference signals from neighboring cells, especially in the case of frequency reuse factor of 1. Various kinds of techniques are required to cope with the interferences, however, among them channel estimation is of great importance because channel state information (CSI) is required by many other interference related techniques, such as interference canceller or adaptive antenna array. Therefore, taking advantage of the specificity of the block transmission schemes with cyclic prefix, we have considered the problem of downlink channel estimation in multi-cell environment.

In this paper, we investigate downlink channel estimation schemes for multi-cell block transmission systems with cyclic prefix and propose a time domain channel estimation scheme using iterative least square (LS) interference cancellation. The analytical derivations of the estimated channel responses of the discrete frequency domain approach and of the time domain approach reveal the fact that the frequency domain approach suffers from interference directly while the time domain approach can reduce the interference power by length of cyclic prefix plus one and the transmitted block (FFT) size. Based on the considerations on the performance improvement by the time domain approach, we propose a time domain channel estimation scheme using iterative interference cancellation. The proposed scheme takes advantage of the fact that each received pilot signal exists in up to the length of the cyclic prefix pulse one dimensional subspace in the FFT size dimensional received signal vector space. By the employment of the iterative procedure, the proposed scheme can obtain LS estimates of not only the channel impulse response between the desired base station and the mobile terminal but also the responses between the interfering base stations and the mobile terminal without calculating pseudoinverse matrices, as far as the same number of matrices as the pilot signal vectors are stored in the mobile terminal. Computer simulation results show that the proposed scheme can achieve almost the same channel estimation accuracy in multi-cell scenario as that in single-cell environment with small number of iterations.

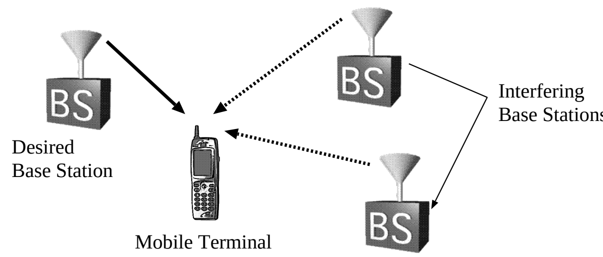

Fig. 1. System Model

II. Signal Modeling

The following notations are used for describing the proposed system. K is the length of the guard interval, M is the FFT size, and L is the channel order. An $M \times M$ identity matrix will be denoted as $\mathbf{I}_M$, a zero matrix of size $A \times B$ will be denoted as $\mathbf{0}_{A\times B}$, and a DFT matrix of size $M \times M$, whose (i, j) element is $\frac{1}{\sqrt{M}} e^{-j\frac{2\pi(i-1)(j-1)}{M}}$, as $\mathbf{D}$. We will use $E[\cdot]$ to denote ensemble average, $(\cdot)^T$ for transpose, $(\cdot)^H$ for Hermitian transpose, and $(\cdot)^*$ for complex conjugate.

Fig. 1 shows the system model considered in this paper. In order to obtain a received signal model at the mobile terminal, we have made following assumptions:

- There are one desired base station and U interfering base stations
- DFT window timings of all the base stations are synchronized
- All base stations use same carrier frequency
- The length of the cyclic prefix K is greater than or equal to the channel order L

With the assumptions above and ignoring additive noise for the simplicity, the received pilot signal vector at the mobile terminal after the cyclic prefix removal can be written as

$$\mathbf{r} = \mathbf{C}_d \mathbf{D}^H \mathbf{p}_d + \sum_{i=1}^{U} \mathbf{C}_{u,i} \mathbf{D}^H \mathbf{p}_{u,i}, \tag{1}$$

where $\mathbf{p}_d = [p_0^d, \ldots, p_{M-1}^d]^T$ denotes a transmitted pilot signal block of the desired base station and $\mathbf{p}_{u,i} = [p_0^{u,i}, \ldots, p_{M-1}^{u,i}]^T$, $(i = 1, \ldots, U)$ is a transmitted pilot signal block from the ith interfering base station. $\mathbf{C}_d$ and $\mathbf{C}_{u,i}$, $(i = 1, \ldots, U)$ are $M \times M$ circulant channel matrices defined as

$$\mathbf{C}_d = \begin{bmatrix} h_0^d & 0 & \ldots & 0 & h_K^d & \ldots & h_1^d \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & h_K^d \\ h_K^d & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \ldots & 0 & h_K^d & \ldots & \ldots & h_0^d \end{bmatrix}, \tag{2}$$

where $\{h_0^d, \ldots, h_K^d\}$ denotes a channel impulse response between the desired base station and the mobile terminal, and

$$\mathbf{C}_{u,i} = \begin{bmatrix} h_0^{u,i} & 0 & \ldots & 0 & h_K^{u,i} & \ldots & h_1^{u,i} \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & h_K^{u,i} \\ h_K^{u,i} & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \ldots & 0 & h_K^{u,i} & \ldots & \ldots & h_0^{u,i} \end{bmatrix}, \tag{3}$$

where $\{h_0^{u,i}, \ldots, h_K^{u,i}\}$, $(i = 1, \ldots, U)$ denotes a channel impulse response between the ith interfering base station and the mobile terminal, respectively. By defining

$$\mathbf{P}_d = [P_0^d, \ldots, P_{M-1}^d]^T = \mathbf{D}^H \mathbf{p}_d, \tag{4}$$

$$\mathbf{P}_{u,i} = [P_0^{u,i}, \ldots, P_{M-1}^{u,i}]^T = \mathbf{D}^H \mathbf{p}_{u,i}, \quad (i = 1, \ldots, U) \tag{5}$$

the received signal block can be simplified as

$$\mathbf{r} = \mathbf{C}_d \mathbf{P}_d + \sum_{i=1}^{U} \mathbf{C}_{u,i} \mathbf{P}_{u,i}. \tag{6}$$

Furthermore, defining impulse response vectors as $\mathbf{h}_d = [h_0^d, \ldots, h_K^d]^T$ and $\mathbf{h}_{u,i} = [h_0^{u,i}, \ldots, h_K^{u,i}]^T$, $(i = 1, \ldots, U)$, and using the property of circulant matrix, we have

$$\mathbf{r} = \mathbf{Q}_d^C \begin{bmatrix} \mathbf{h}_d \\ \mathbf{0}_{(M-K-1)\times 1} \end{bmatrix} + \sum_{i=1}^{U} \mathbf{Q}_{u,i}^C \begin{bmatrix} \mathbf{h}_{u,i} \\ \mathbf{0}_{(M-K-1)\times 1} \end{bmatrix}, \tag{7}$$

where

$$\mathbf{Q}_d^C = \begin{bmatrix} P_0^d & P_{M-1}^d & \ldots & P_1^d \\ P_1^d & P_0^d & \ddots & \vdots \\ \vdots & \ddots & \ddots & P_{M-1}^d \\ P_{M-1}^d & \ldots & P_1^d & P_0^d \end{bmatrix}, \tag{8}$$

$$\mathbf{Q}_{u,i}^C = \begin{bmatrix} P_0^{u,i} & P_{M-1}^{u,i} & \ldots & P_1^{u,i} \\ P_1^{u,i} & P_0^{u,i} & \ddots & \vdots \\ \vdots & \ddots & \ddots & P_{M-1}^{u,i} \\ P_{M-1}^{u,i} & \ldots & P_1^{u,i} & P_0^{u,i} \end{bmatrix}. \tag{9}$$

Finally, defining matrices $\mathbf{Q}_d$ and $\mathbf{Q}_{u,i}$, $(i = 1, \ldots, U)$ to be the first $K+1$ columns of $\mathbf{Q}_d^C$ and $\mathbf{Q}_{u,i}^C$, respectively, we have

$$\mathbf{r} = \mathbf{Q}_d \mathbf{h}_d + \sum_{i=1}^{U} \mathbf{Q}_{u,i} \mathbf{h}_{u,i}. \tag{10}$$

Note that we have assumed OFDM access (OFDMA) signaling so far and will assume hereafter, however, all the following discussions can be applied to the SC-CP systems by just setting the vectors $\mathbf{P}_d$ and $\mathbf{P}_{u,i}$, $(i = 1, \ldots, U)$ to be the transmitted pilot signal vectors.

III. Downlink Channel Estimation

Before we show the proposed downlink channel estimation scheme, we firstly review two conventional channel estimation schemes, namely, discrete frequency domain channel estimation approach and time domain approach, and derive residual interference signals in the estimated channel responses.

A. Discrete frequency domain approach

In order to estimate channel frequency response, we perform DFT of the received pilot signal of Eq. (1) as

$$\mathbf{D}\mathbf{r} = \mathbf{D}\mathbf{C}_d \mathbf{D}^H \mathbf{p}_d + \sum_{i=1}^{U} \mathbf{D}\mathbf{C}_{u,i} \mathbf{D}^H \mathbf{p}_{u,i}, \tag{11}$$

$$= \mathbf{\Lambda}_d \mathbf{p}_d + \sum_{i=1}^{U} \mathbf{\Lambda}_{u,i} \mathbf{p}_{u,i}, \tag{12}$$

where $\mathbf{\Lambda}_d = \text{diag}[\lambda_0^d, \ldots, \lambda_{M-1}^d]$ and $\mathbf{\Lambda}_{u,i} = \text{diag}[\lambda_0^{u,i}, \ldots, \lambda_{M-1}^{u,i}]$, $(i = 1, \ldots, U)$ are diagonal matrices whose diagonal elements are calculated as [6]

$$\begin{bmatrix} \lambda_0^d \\ \vdots \\ \lambda_{M-1}^d \end{bmatrix} = \mathbf{D} \begin{bmatrix} \mathbf{h}_d \\ \mathbf{0}_{(M-K-1)\times 1} \end{bmatrix}, \tag{13}$$

$$\begin{bmatrix} \lambda_0^{u,i} \\ \vdots \\ \lambda_{M-1}^{u,i} \end{bmatrix} = \mathbf{D} \begin{bmatrix} \mathbf{h}_{u,i} \\ \mathbf{0}_{(M-K-1)\times 1} \end{bmatrix}. \tag{14}$$

Therefore, the mth frequency response of the channel between the desired base station and the mobile terminal λ_m^d can be estimated as

$$\hat{\lambda}_m^d = \frac{\{\mathbf{Dr}\}_m}{p_m^d} = \lambda_m^d + \sum_{i=1}^{U} \frac{p_m^{u,i}}{p_m^d} \lambda_m^{u,i}. \tag{15}$$

From Eq. (15), we can see that the second term of right hand side becomes residual interferences on the estimated frequency response. If we assume the same statistical property for all the channels, each pilot signal from the interfering base station results in direct interference for the desired frequency response. Although, in practical situations, the amplitude of $\lambda_m^{u,i}$ will be smaller than that of λ_m^d on average, we can not expect accurate channel estimation by the discrete frequency domain approach, especially for the case of frequency reuse factor of 1.

B. Time domain approach

The time domain channel estimation scheme is based on LS criterion. Defining Moore-Penrose generalized inverse of $\mathbf{Q}_d$ as

$$\mathbf{Q}_d^+ = (\mathbf{Q}_d^H \mathbf{Q}_d)^{-1} \mathbf{Q}_d^H, \tag{16}$$

the channel impulse response between the desired base station and the mobile terminal $\mathbf{h}_d$ can be estimated as

$$\hat{\mathbf{h}}_d = \mathbf{Q}_d^+ \mathbf{r} = \mathbf{h}_d + \sum_{i=1}^{U} \mathbf{Q}_d^+ \mathbf{Q}_{u,i} \mathbf{h}_{u,i}. \tag{17}$$

In Eq. (17), the second term of the right hand side is also the residual interference, however, unlike the discrete frequency domain approach, we can expect a certain reduction of the interferences in this case. The multiplication of the pseudoinverse of $\mathbf{Q}_d$ extracts elements in the $K+1$ dimensional vector subspace, which is spanned by the column vectors of $\mathbf{Q}_d$, therefore, the powers of the interferences are reduced by the pseudoinverse as far as the column spaces of $\mathbf{Q}_d$ and $\mathbf{Q}_{u,i}$ are not exactly the same.

Fig. 2 shows a geometric explanation of the principle of the improvement, where $U = 1$, $M = 2$ and $K = 0$ are employed and $S(\mathbf{Q}_d)$, $S(\mathbf{Q}_{u,1})$ and $Ker(\mathbf{Q}_d^+)$ denote the column spaces of $\mathbf{Q}_d$ and $\mathbf{Q}_{u,i}$, and the kernel space of $\mathbf{Q}_d^+$, respectively. As we can see from the figure, the degree of the reduction of the interference depends on the relation between the column spaces of $S(\mathbf{Q}_d)$ and $S(\mathbf{Q}_{u,1})$. For example, if $\mathbf{p}_d$ and $\mathbf{p}_{u,i}$ are independently and randomly determined, the probability of overlapping the column spaces is $(K+1)/M$, therefore, each interference will be reduced by $(K+1)/M$ on average. This will be confirmed via computer simulations.

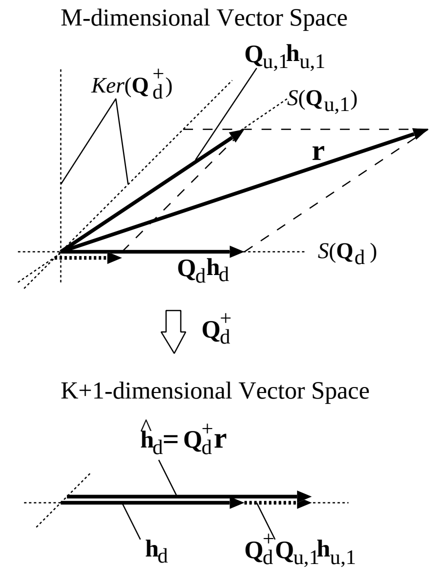

Fig. 2. Geometric Explanation of Time Domain Approach

C. Time domain approach with iterative interference canceller

Base on the discussions above, we can further improve the accuracy of the channel estimation if the pilot signals of the interfering base stations are available at the mobile terminal. The assumption of the availability of the pilot signals at the mobile terminal is valid in general because the pilot signals can be specified using base station identification numbers detected by the mobile terminal.

From Eq. (7), we can see that the received signal model has a symmetric structure with respect to the pilot signal components from the desired base station and the interfering base stations. This means that the channel impulse response between the interfering base stations and the mobile terminal $\mathbf{h}_{u,i}$, $(i = 1, \ldots, U)$ can be estimated in the same way as Eq. (17), therefore, the estimated response will be

$$\hat{\mathbf{h}}_{u,i} = \mathbf{Q}_{u,i}^+ \mathbf{r} = \mathbf{h}_{u,i} + \mathbf{Q}_{u,i}^+ \mathbf{Q}_d \mathbf{h}_d + \sum_{\substack{j=1 \\ j \neq i}}^{U} \mathbf{Q}_{u,i}^+ \mathbf{Q}_{u,j} \mathbf{h}_{u,j}, \tag{18}$$

where

$$\mathbf{Q}_{u,i}^+ = (\mathbf{Q}_{u,i}^H \mathbf{Q}_{u,i})^{-1} \mathbf{Q}_{u,i}^H. \tag{19}$$

Using $\hat{\mathbf{h}}_{u,i}$, $(i = 1, \ldots, U)$ for the interference cancellation, the improved channel estimation can be obtained as

$$\hat{\hat{\mathbf{h}}}_d = \mathbf{Q}_d^+ (\mathbf{r} - \sum_{i=1}^{U} \mathbf{Q}_{u,i} \hat{\mathbf{h}}_{u,i}). \tag{20}$$

Fig. 3 shows a geometric explanation of the principle of the interference cancellation, where $U = 1$, $M = 2$ and $K = 0$ are employed. By the cancellation process, the residual interference in the estimated channel response can be reduced from $\mathbf{Q}_d^+ \mathbf{Q}_{u,1} \mathbf{h}_{u,1}$ to $\mathbf{Q}_d^+ \mathbf{Q}_{u,1} \mathbf{Q}_{u,1}^+ \mathbf{Q}_d \mathbf{h}_d$. As we can see

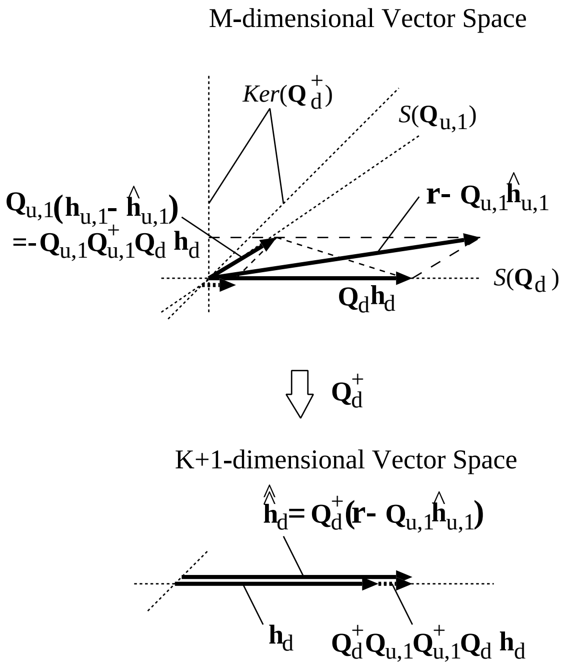

Fig. 3. Geometric Explanation of Time Domain Approach with Interference Canceller

from the figure, since the improvement of the estimate of $\mathbf{h}_d$ results in the improved estimate of $\mathbf{h}_{u,i}$, and vice versa, more accurate channel estimation can be achieved through an iterative cancellation and estimation procedure.

Denoting estimated responses between the desired base station and the mobile terminal and between the ith interfering base station and the mobile terminal at the kth iteration as $\hat{\mathbf{h}}_d^k$ and $\hat{\mathbf{h}}_{u,i}^k$, $(i = 1, \ldots, U)$, respectively, the proposed iterative channel estimation scheme is given as follows:

1) $k = 0$
2) Initial estimation of $\mathbf{h}_d$:

$$\hat{\mathbf{h}}_d^k = \mathbf{Q}_d^+ \mathbf{r}$$

3) Initial estimation of $\mathbf{h}_{u,i}$, $(i = 1, \ldots, U)$:

$$\hat{\mathbf{h}}_{u,i}^k = \mathbf{Q}_{u,i}^+ \mathbf{r}$$

4) $k = k + 1$
5) Estimation of $\mathbf{h}_d$:

$$\hat{\mathbf{h}}_d^k = \mathbf{Q}_d^+ (\mathbf{r} - \sum_{i=1}^{U} \mathbf{Q}_{u,i} \hat{\mathbf{h}}_{u,i}^{k-1})$$

6) Estimation of $\mathbf{h}_{u,i}$, $(i = 1, \ldots, U)$:

$$\hat{\mathbf{h}}_{u,i}^k = \mathbf{Q}_{u,i}^+ \left(\mathbf{r} - \mathbf{Q}_d \hat{\mathbf{h}}_d^{k-1} - \sum_{\substack{j=1 \\ j \neq i}}^{U} \mathbf{Q}_{u,j} \hat{\mathbf{h}}_{u,j}^{k-1} \right)$$

7) Go to step 4

Note that the proposed method requires the pseudoinverse of the matrices, which requires high computational complexity in general, however, the computation of the pseudoinverse can be avoided if we store the pseudoinverse matrices in the mobile terminal. Since there is one-to-one relationship between the pilot signal vector and the pseudoinverse matrix, the number of matrices to be stored in the mobile terminal is the same as the number of the pilot signal vectors.

Note also that we can achieve the same accuracy of the channel estimation as the proposed method above without using iterative calculation. Since the received pilot signal vector can be modified as

$$\begin{aligned} \mathbf{r} &= \mathbf{Q}_d \mathbf{h}_d + \sum_{i=1}^{U} \mathbf{Q}_{u,i} \mathbf{h}_{u,i} \\ &= \begin{bmatrix} \mathbf{Q}_d & \mathbf{Q}_{u,1} & \cdots & \mathbf{Q}_{u,U} \end{bmatrix} \begin{bmatrix} \mathbf{h}_d \\ \mathbf{h}_{u,1} \\ \vdots \\ \mathbf{h}_{u,U} \end{bmatrix}, \end{aligned} \tag{21}$$

the LS estimate of all the impulse responses can be obtained in one step by multiplying the pseudoinverse of $[\mathbf{Q}_d, \mathbf{Q}_{u,1}, \ldots, \mathbf{Q}_{u,U}]$ from the left. The idea itself of the one step approach is quite simple, however, it requires huge amount of memory to store the pseudoinverse matrices of all possible combinations or high computational complexity due to the calculation of the pseudoinverse. After all, it can be concluded that the proposed iterative channel estimation scheme is much more feasible than the one step approach.

IV. Computer Simulation

In order to confirm the performance of the proposed method, computer simulations are conducted with the following system parameters; Mod./Demod. scheme: QPSK coherent detection, symbols per block: $M = 128$, length of cyclic prefix: $K = 16$, channel order: $L = 16$, channel model: 17-path frequency selective Rayleigh fading channel, number of interfering base stations: $U = 1$, channel noise: additive white Gaussian noise (AWGN). Also, the pilot signal vectors of the desired base station and the interfering base stations are randomly determined.

Figs. 4 and 5 show the normalized mean-square-error (MSE) of the estimated channel between the desired base station and the mobile terminal versus the number of iterations of the interference cancellation $k = 0, \ldots, 5$ of the proposed channel estimation scheme for OFDMA and SC-CP systems, respectively. The normalized MSE is defined as

$$(\text{Normalized MSE}) = \frac{1}{N_{trial}} \sum_{m=1}^{N_{trial}} \frac{||\mathbf{h}_d - \hat{\mathbf{h}}_d^k||^2}{||\mathbf{h}_d||^2}, \tag{22}$$

where N_{trial} denotes the number of channel realizations and has been set to be 1,000 and $|| \cdot ||$ is an Euclidian norm. For comparison purpose, the performances of the time domain channel estimation without the interference and those of the proposed scheme without channel noise are also plotted in

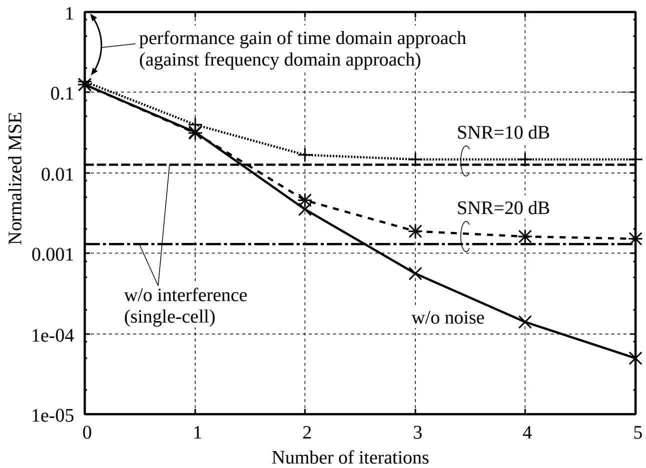

Fig. 4. Normalized MSE of Channel Estimation: OFDMA

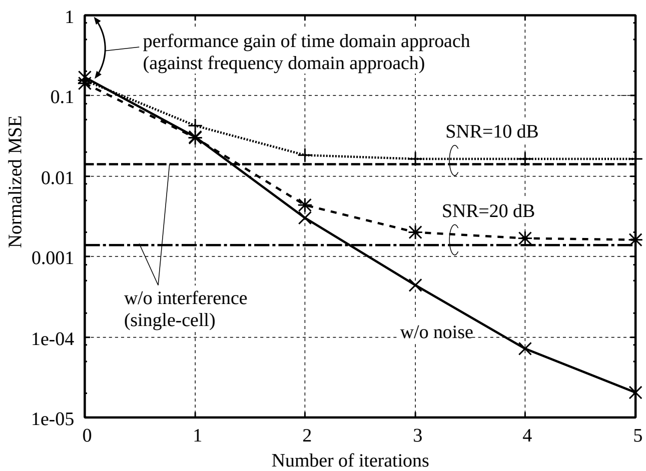

Fig. 5. Normalized MSE of Channel Estimation: SC-CP

the same figures. From the figures, we can see that the proposed channel estimation scheme can achieve almost the same accuracy as the time domain channel estimation scheme without interference signal, namely single-cell environment, in 4 iterations for both of the systems. Since the normalized MSE of the discrete frequency domain approach is equal to 1 in the computer simulation settings, the performance gains obtained by the time domain approach are the indicated parts in the figures, which are about 9 dB. This is because the probability of overlapping the column spaces of $\mathbf{Q}_d$ and $\mathbf{Q}_{u,1}$ is equal to $(K+1)/M \approx 1/8 \approx -9$ dB due to the random generation of the pilot signal vectors. Note that the proposed scheme does not require perfect orthogonality among the pilot signal vectors of the desired base station and the interfering base stations, and can also obtain the estimate of the channel response between the interfering base station and the mobile terminal with the same accuracy as that of the channel between the desired base station and the mobile terminal in the iterative channel estimation process.

V. Conclusion

We have considered downlink channel estimation schemes for multi-cell block transmission systems with cyclic prefix. It has been analytically shown that the frequency domain approach suffers from interference directly while the time domain approach can reduce the interference power by the ratio of the length of cyclic prefix plus one and the FFT size $(K+1)/M$. Based on the analysis, we have proposed a time domain channel estimation scheme using iterative LS interference cancellation, where the iterative procedure enables us to have the LS estimates of both the channel response between the desired base station and the mobile terminal and between the interfering base stations and the mobile terminal, simultaneously. From the computer simulation results, it could be concluded that the proposed scheme in multi-cell scenario can achieve almost the same estimation accuracy as the LS time domain channel estimation in single-cell environment.

Future work may include interference mitigation techniques for multi-cell block transmission systems with cyclic prefix based on the estimated channel responses using the proposed scheme.

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