Power Allocation Scheme for MIMO MC-CDMA With Two Dimensional Spreading

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***Abstract*— In this paper, we develop a power distribution scheme for multi-input multi-output (MIMO) [1] multi-carrier (MC) code division multiple access (CDMA) systems [2]-[3] with two-dimensional spreading for wireless transmission. We propose to allocate power among symbols of the active users transmitted from multiple transmit antennas based on a heuristic expression of the bit error rate (BER). Optimization principle consists of minimizing the average BER of all users in term of channel conditions, user multiplexing and number of transmit antennas. Due to the simplicity of the approximate BER expression, we can obtain a closed form expression of the optimum power to be allocated to each user on each transmit antenna. Simulation results show significant improvement of the BER performances compared to the equal power distribution for various modulations, several encoding gains and two antenna configurations.**

***Keywords*- Two-dimensional spreading, MC-CDMA, Lagrangian method, global BER optimization.**

I. Introduction

The combination of code division multiple access (CDMA) signal processing with orthogonal frequency division multiplexing (OFDM) using cyclic prefix (CP) and multiple antennas for downlink transmission [4] is regarded as a highly promising solution to achieving high data rates of next-generation wireless communication systems operating in frequency selective fading and multi-cell environments. Recently, several authors have proposed two dimensional spreading for MC-CDMA transmission to take advantages of the cell environment [5]-[6]. It employs the total spreading factor $SF = SF_f \times SF_t$ of greater than 1 in a multi-cell environment to achieve higher link capacity [7]-[8]. SF_t and SF_f represent the spreading values in the time and frequency domains, respectively. This is because one-cell frequency reuse is possible for $SF > 1$ by introducing a cell specific scrambling code and it is expected a direct increase in the radio link capacity by employing sectorization. The time and frequency domain spreading MC-CDMA broadband access employs the two-layered spreading by the cell-specific scrambling code and cell specific orthogonal code. In this paper, we propose a novel power allocation scheme which takes advantage of the channel condition, the antenna configuration and the number of active users. The proposed principle consists of minimizing the average BER of all users in term of channel conditions, user multiplexing and number of transmit antennas. Optimization principle consists of minimizing the average BER of all users in term of channel conditions, user multiplexing and number of transmit antennas. Due to the simplicity of the approximate BER expression, we can obtain a closed form expression of the optimum power to be allocated to each user on each transmit antenna.

The layout of the paper is as follows. In Section II we review the system model. In Section III, we introduce the estimate value of BER for different channel encoded sequences and we describe in detail the proposed power adaptation scheme. Section IV gives the experimental results over QPSK and QAM modulations for two different multiple antenna configurations. Finally, conclusions are drawn in Section V.

II. System Description

The generation of a two-dimensional spreading MC-CDMA signal with multiple antennas can be described as follows [9]-[10]. First, for each active user and each transmit antenna, the binary data are encoded (convolution coding or turbo coding), modulated (PSK or QAM modulation), and the single data symbol is replicated into parallel copies. In the two-dimensional spreading, the operation of time-domain spreading is that a modulated symbol is copied into consecutive SF_t MC-CDMA elements and each of the copied symbols is multiplied by the corresponding chip of the spreading code with SF_t. The operation of frequency-domain spreading is that a data modulated symbol is copied into consecutive SF_f MC-CDMA elements and each of the copied symbols is multiplied by the corresponding chip of the spreading code with SF_f. When the number of MC-CDMA elements in a packet frame and the number of sub-carriers are represented by N_d and N, respectively, the resultant two-dimensional spreading sequence of the v-th MC-CDMA symbol ($v = 0, 1, ..., N_d - 1$) at the w-th subcarrier ($w = 0, 1, ..., N - 1$) for the l-th transmit antenna ($l = 0, ..., N_t - 1$) is expressed as:

$$x_{v,w}^{(l,k)} = \sqrt{p_{k,\lfloor w/SFf \rfloor}^{l}} \cdot d_{v/SFt,\lfloor w/SFf \rfloor}^{(l,k)} \cdot c_{(v \mod SFt),(w \mod SFf)}^{(k)} \quad (1)$$

where $d_{i,j}^{(l,k)}$ $(i = 0, 1, ..., N_d/SF_t, j = 0, 1, ..., N/SF_f)$ is the data modulated symbol on the l-th transmit antenna for the k-th active user, $c_{i,j}^{(k)}$ $(i = 0, 1, ..., SF_t, j = 0, 1, ..., SF_f)$ is the two-dimensional spreading code sequence for the k-th active user and $p_{k,j}^{l}$ is the allocated power of the k-th active user for the j-th modulated symbol on the l-th transmit antenna. In the case of equal power distribution, the term $p_{k,j}^{l}$ becomes equal to the constant average transmit transmit power which is denoted $\overline{P}$ in the rest of the paper. Furthermore, $\lfloor z \rfloor$ denotes the greatest integer that does not exceed z. Note that the number of data modulated symbols within a packet frame is represented by $(N_t \cdot N \cdot N_d)/(SF_f \cdot SF_t)$.

For each transmit antenna, spreading sequence of the v-th MC-CDMA symbol at the w-th subcarrier for the l-th transmitted antenna is given by

$$x_{v,w}^{mux(l)} = \sum_{k=0}^{N_u-1} x_{v,w}^{(l,k)}, \tag{2}$$

where N_u is the number of active users.

The modulated signals are frequency-division multiplexed by an inverse discrete Fourier transform (IDFT) [11]. The resultant symbol sequence is converted into MC-CDMA signals $(T_d = N \cdot T_{fft})$ where T_{fft} represents the duration of IDFT sample. In order to avoid inter-symbol interference (ISI) caused by multi-path propagation, a guard interval with N_g points $(T_g = N_g \cdot T_{fft})$ is inserted between the MC-CDMA symbols. After the GI addition, the MC-CDMA signal is equal to:

$$\begin{aligned} s^l(t) = \sum_{v=0}^{N_d-1} \sum_{w=0}^{N-1} x_{v,w}^{mux(l)} \cdot u(t - v(Td + Tg)) \\ \cdot \exp\left\{ j \frac{2\pi w(t - v(T_d + T_g))}{T_d} \right\} \end{aligned} \tag{3}$$

where the term u(t) is the step function defined within the duration of $[0, T_d + T_g)$. The signal $s^l(t)$ is then transmitted through a multi-path wireless channel characterized to be a slowly varying Rayleigh fading channel for each subcarrier [12]-[13], with the transfer function $\xi_{n,l,m}^{(k)} = \alpha_{n,l,m}^{(k)} \cdot \exp(j\varphi_{n,l,m}^{(k)})$, for $l = 0, ..., N_t$, and $n = 0, ..., N_r$. N_r represents the number of receive antennas. We assume that $\alpha_{n,l,m}^{(k)}$ and $\varphi_{n,l,m}^{(k)}$ are respectively i.i.d. Rayleigh random variables with a unit moment, and uniform random variables over $[0, 2\pi)$ for the transmit and receive antennas [14].

At the receiving part for each user on each receive antenna, after removing the GI part, the received MC-CDMA signal is sampled into N orthogonal subcarriers by applying the discrete Fourier transform (DFT) [15]. The received component of the v-th MC-CDMA element at the w-th subcarrier is expressed by:

$$\begin{aligned} Y_{v,w}^{(n)} = \frac{1}{T_d} \cdot \int_{v.(T_d+T_g)}^{T_d+v.(T_d+T_g)} r^{(n)}(t) \\ \cdot \exp\left\{ -j \frac{2\pi w.(t - v(T_d + T_g))}{T_d} \right\} \cdot dt \end{aligned} \tag{4}$$

In Eq. 4, $r^{(n)}(t)$ denotes the time domain representation of the received signal on the n-th receive antenna [16]. Including the channel response, we obtain for the specific k-th active user:

$$Y_{v,w}^{(n)} = \sum_{l=0}^{Nt-1} \xi_{n,l,w}^{(k)} . x_{v,w}^{(l)} + e_{n,v,w}^{(k)} \tag{5}$$

where $e_{n,v,w}^{(k)}$ denotes the additive white Gaussian noise (AWGN) of variance σ_n^2 on the n-th received antenna for the k-th active user. Because of the channel fading, the channel frequency response is not constant over the transmit bandwidth and the orthogonality between the different users may be destroyed. To mitigate the effect of the channel distortion, various detection schemes such as zero forcing (ZF) scheme or minimum mean square error (MMSE) scheme, can be applied on the received signal in the frequency domain. Finally, to extract, from the received signal, specific data of one user, despreading step is performed and data, over the interval of chips are coherently accumulated in both the frequency and the time domains. The despread data sequence of the k-th code channel (i.e. active user), $\hat{d}_{i,m}^{(l,k)}$ $(i = 0, ..., N_d/SF_t, m = 0, ..., N/SF_f)$ is represented as:

$$\begin{aligned} \hat{d}_{i,m}^{(l,k)} = \frac{1}{SF} . \sum_{v=0}^{SF_t-1} \sum_{w=0}^{SF_f-1} \sum_{n=0}^{N_r-1} G_{w+SF_f \cdot m}^{(k,l,n)} \\ \cdot Y_{i \cdot SF_t+v, w+SF_f \cdot m}^{(n)} \cdot c_{v,w}^{(k)} \end{aligned} \tag{6}$$

where $G_{w+SF_f \cdot m}^{(k,l,n)}$ is the channel equalizer coefficient for the k-th active user on the $(w + SF_f \cdot m)$-th subcarrier between the l-th transmit and the n-th receive antennas.

III. Power Allocation Scheme

A. Bit Error Rate approximation

We next list the assumptions adopted in this paper. The channel is frequency selective fading, and remains invariant per frame, but is allowed to vary from frame to frame. This corresponds to a block fading channel model, which is suitable for slowly-varying channels. As a consequence, power allocation is adjusted on a frame-by-frame basis. Perfect channel state information (CSI) is available at the receiver and the corresponding CSI is fed back to the transmitter without any error and latency. The assumption that the feedback channel is error free and has no latency, could be at least approximatively satisfied by using a fast feedback link with powerful error control for feedback information. Further considerations on system design with delayed or noisy CSI will be in the future considered.

For fading channel adhering to the previous assumptions, the channel characteristics is captured by the received SNR. The received SNR is a function of the variance of the noise, the channel state information, and the transmit power. Since the channel varies from frame by frame, the approximated BER can be described as a function of the received SNR (i.e. transmit power for each transmit antenna and channel condition). Recently, several authors [18]-[19] proposed heuristic equations to estimate for the case of ZF detection scheme, the approximation of the BER as:

$$f(\beta_{k,m}^l, p_{k,m}^l) \approx a \cdot \exp\{-b \cdot \beta_{k,m}^l \cdot p_{k,m}^l\}, \quad (7)$$

$$\beta_{k,m}^l = \frac{SF_f}{(2^{N_m}-1)\cdot \sigma_n^2 \cdot \sum_{n=0}^{Nr-1}\sum_{q=0}^{SF_f-1} |G_{q+SF_f\cdot m}^{(k,l,n)}|^2}, \quad (8)$$

where a and b are heuristic parameters to be evaluated and N_m is the number of bits per modulated symbol. From Eqs. (7) and (8), the information to be fed back from the receiver to the transmitter in order to estimate the power distribution is limited to the term:
$\sum_{q=0}^{SF_f-1} |G_{q+SF_f\cdot m}^{(k)}|^2$,
In this paper, we consider the following two transmission schemes [19]:
- Uncoded: Without forward error correction (FEC) with QPSK and QAM modulations.
- Convolutionally coded: With convolutional code used in this paper, is based on the description proposed in [20]-[21]. The generator polynomial of the mother code is $g = [133, 171]$ and the coding rates are obtained from the puncturing pattern described in [20]. Tables I, II and III summarize the heuristic value of the parameters *a* and *b* used in (7) for several coding rates and several modulations which are obtained by fitting with computer simulations results, especially for the BER range between 10^{-2} and 10^{-5}.

TABLE I
TRANSMISSION MODES FOR QPSK MODULATION

Modulation	QPSK	QPSK	QPSK
Coding Rate	1/2	3/4	1
Rate (bits/symb.)	1	1.5	2
a	7	16	0.2
b	9.5	5.4	1.66

TABLE II
TRANSMISSION MODES FOR 16-QAM MODULATION

Modulation	16-QAM	16-QAM	16-QAM
Coding Rate	1/2	3/4	1
Rate (bits/symb.)	2	3	4
a	4	14	0.2
b	11	6	1.73

TABLE III
TRANSMISSION MODES FOR 64-QAM MODULATION

Modulation	64-QAM	64-QAM	64-QAM
Coding Rate	1/2	3/4	1
Rate (bits/symb.)	3	4.5	6
a	1.5	7	0.15
b	12	6	1.68

B. Power adaptation

The basic principle of the proposed power allocation for two-dimensional spreading MC-CDMA signal with multiple antennas is to obtain the transmit power distribution such that minimizes average BER of all the active users by utilizing the approximated BER expression. Moreover, constraint that the maximum total transmit power is imposed [22]. In addition, the spreading gain issue from the different orthogonal codes is included in the approximate BER expression.
The global power resource assigned by the proposed algorithm satisfies the simple relation that the total transmit power is kept constant for all the transmit antenna and all the active users. This constraint for the m-th modulated element ($m = 0, ..., N/SF_f$) is given by:

$$\sum_{l=0}^{N_t-1}\sum_{k=0}^{N_u-1} p_{k,m}^l = N_t \cdot N_u \cdot \overline{P} \quad (9)$$

where $\overline{P}$ is the constant average transmit power.
The average BER becomes minimal when the BER is minimized for each given channel state. An equivalent mathematical representation of the optimization problem for the m-th modulated element is described by:

$$\begin{cases} \min \frac{1}{N_t.N_u}\sum_{l=0}^{N_t-1}\sum_{k=0}^{N_u-1} f(\beta_{k,m}^l, p_{k,m}^l) \\ \text{subject to:} \\ \sum_{l=0}^{N_t-1}\sum_{k=0}^{N_u-1} p_{k,m}^l = N_t \cdot N_u \cdot \overline{P} \end{cases} \quad (10)$$

One possibility to solve this optimization problem is to apply the Lagrangian procedure. Defining Lagrangian as:

$$J = \frac{1}{N_t \cdot N_u}\sum_{l=0}^{N_t-1}\sum_{k=0}^{N_u-1} f(\beta_{k,m}^l, p_{k,m}^l) + \lambda \cdot (\sum_{l=0}^{N_t-1}\sum_{k=0}^{N_u-1} p_{k,m}^l - N_t \cdot N_u \cdot \overline{P}), \quad (11)$$

We finally obtain the following analytical solution:

$$p_{k,m}^l = \left[\sum_{\alpha=0}^{N_t-1}\sum_{\gamma=0}^{N_u-1} \frac{\beta_{k,m}^l}{\beta_{\gamma,m}^\alpha}\right]^{-1} \cdot \Bigg[N_t.N_u.\overline{P} + \sum_{\alpha=0}^{N_t-1}\sum_{\gamma=0}^{N_u-1} \frac{1}{b.\beta_{\gamma,m}^\alpha} \cdot \log\left(\frac{\beta_{k,m}^l}{\beta_{\gamma,m}^\alpha}\right)\Bigg] \quad (12)$$

Since we have ignored the range of $p_{k,m}$ so far for the simplicity, the Lagrangian procedure sometimes results in negative value. In this case we propose to use the equal power distribution scheme.

IV. EXPERIMENTATION

We now evaluate the performance of the proposed power allocation for the two-dimensional spreading MC-CDMA scheme with multiple antenna in a multi-path fading environment. Main simulation parameters are presented in Table IV. We assume perfect knowledge of the channel conditions both at the transmitting and receiving parts. The effect of the number of transmit antennas and the coding rates are highlighted in the simulation results presented in this section. In all the simulations, conventional scheme means equal power distribution between the different active users and the transmit antennas and it is set-up to $\overline{P}$.

TABLE IV
SIMULATION PARAMETERS

Carrier Frequency	2 GHz
Bandwidth	20 MHz
Modulations	QPSK, 16-QAM, 64-QAM
Spreading configuration	$SF_t = 4$, $SF_f = 4$
Number of active users	$N_u = 12$
Channel Coding	$R = 1$, $R = 1/2$, $R = 3/4$
Channel estimation	Perfect CSI
Number of data subcarrier	128
Guard Interval length	32
Channel model	10-path, Rayleigh Fading
(N_t, N_r)	(2,2) and (4,4)
Number of data packet	40

A. MIMO MC-CDMA without channel coding

Figs. 1 and 2 show the BER versus the total SNR in dB for the specific case of two different spatial configurations for the QPSK, 16-QAM and 64-QAM modulations. Both equal power distribution and proposed schemes are plotted in these figures and results are presented for ZF detection scheme. Here, we have evaluated the performance of the uncoded case.
Fig. 1 highlights the improvement obtained by performing the proposed power allocation scheme compared to the equal power distribution for two transmit and two receive antennas $(N_t = N_r = 2)$. At BER=10^{-4}, respectively 7dB, 6.5dB and 6dB gains for QPSK, 16-QAM and 64-QAM modulations are obtained. At BER=10^{-3}, an average gain of 5dB is obtained for all the modulations schemes. For low SNR, we can see that the equal power allocation scheme outperforms the proposed scheme. This is because we have determined the heuristic parameters of *a* and *b* for the BER range between 10^{-2} and 10^{-5}. By just changing the fitting range of the parameters, the proposed scheme can improve the performance of any required BER range.

In Fig. 2, simulation results are plotted for the specific configuration $N_t = 4$ and $N_r = 4$. This figure shows that at average BER=10^{-4}, respectively, 5.5dB, 5dB, and 4.5dB gains are obtained for the QPSK, 16-QAM and 64-QAM modulations.

B. MIMO MC-CDMA with channel coding

Fig. 3 shows the computer simulation results for channel coding rate $R = 1/2$, and the spatial configuration $N_t = N_r = 4$. Gains obtained by the proposed power allocation scheme are equal to 5dB for QPSK odulation, 4dB

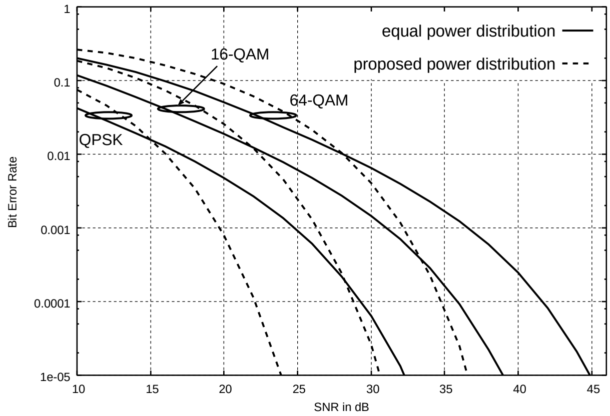

Fig. 1. Performance simulations for $N_t = N_r = 2$ and $R = 1$

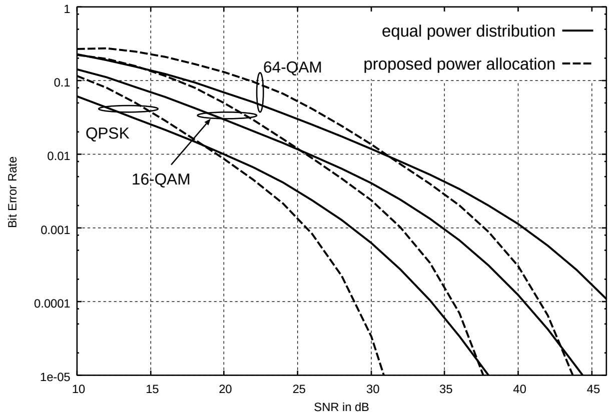

Fig. 2. Performance simulations for $N_t = N_r = 4$ and $R = 1$

for 16-QAM modulation and 3.8dB for 64-QAM modulation at average BER=10^{-4}.

Fig. 4 is for the coding gain $R = 3/4$ and the spatial case $N_t = N_r = 4$. Proposed power distribution scheme outperforms the equal power distribution scheme from SNR=22dB for QPSK modulation, SNR=25dB for 16-QAM modulation and SNR=28dB for 64-QAM modulation.

V. CONCLUSION

In this paper, we have presented an original method to distribute the power allocation between spreading code and spatial domains for MIMO MC-CDMA transmission with two-dimensional spreading. The basic procedure consists of allocating transmit power in function of the channel conditions and the global BER requirement. The simulation results have shown promising results in term of BER for several sets of antenna configuration and three different modulations. Future works would include the introduction of the error in the channel estimation due to the feedback of the CSI [23]-[24]. In addition, this proposed system would be extended to any type of powerful detection scheme such as QRM-MLD [25] and any type of coding scheme such as

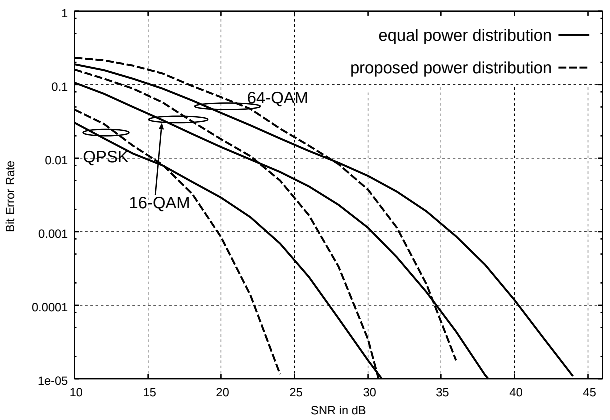

Fig. 3. Performance simulations for $N_t = N_r = 4$ and $R = 1/2$

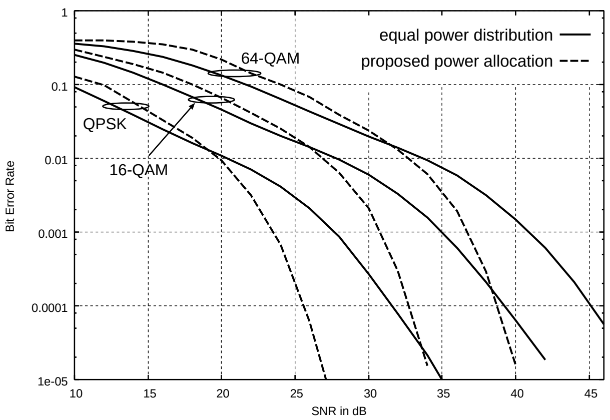

Fig. 4. Performance simulations for $N_t = N_r = 4$ and $R = 3/4$

turbo code [26] or low density parity code [27]-[28].

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