A Subtractive Interference Cancellation Scheme for Single Carrier Block Transmission with Insufficient Cyclic Prefix

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Abstract– **We propose a simple inter-symbol interference (ISI) and inter-block interference (IBI) cancellation scheme for single carrier block transmission with cyclic prefix (SC-CP) systems with insufficient guard interval (GI). In the SC-CP system, the equalization and demodulation processing is conducted in a block-by-block manner, therefore, the IBI can be reduced by using previously detected data signals. For the ISI cancellation, we firstly generate replica signal of the ISI using tentative decisions in order to make the defective channel matrix to be circulant, and then we perform the conventional FDE to compensate the ISI. We also derive minimum mean–square–error (MMSE) and zero forcing (ZF) equalizers for the shake of performance benchmark. Computer simulation results show that the proposed interference cancellation scheme can significantly improve the bit error rate (BER) performance and can outperform the MMSE equalizer while it requires lower computational complexity.**

Key Words: block transmission, cyclic prefix, guard interval, inter-block interference, inter-symbol interference

1. INTRODUCTION

A block transmission with cyclic prefix (CP), including orthogonal frequency division multiplexing (OFDM)[1] and single carrier block transmission with cyclic prefix (SC-CP)[2], has been drawing much attention due to the effective and simple frequency domain equalizer (FDE) using fast Fourier transform (FFT). If all the delayed signals exist within a guard interval (GI), the insertion of the CP as the GI at the transmitter and the removal at the receiver eliminates inter-block interference (IBI). Moreover, the CP operation converts a Toeplitz channel matrix into a circulant matrix, therefore, the inter-symbol interference (ISI) can be effectively equalized by the FDE.

On the other hand, delayed signals beyond the GI deteriorate the performance of the block transmission with the CP. This is because, with the delayed signals, the IBI can not be eliminated by the CP removal and the channel matrix is no more the circulant matrix. So far, a considerable number of studies have been made on the issue, such as impulse response shortening[3], utilization of an adaptive antenna array[4] and per-tone equalization[5],[6]. All the methods can improve the performance, however, they increase the computational or system complexity, which may spoil the important feature of the FDE based systems. On the contrary, the interference elimination scheme proposed in [7] is quite simple, however, it reduces the transmission rate.

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In this paper, we propose a simple ISI and IBI cancellation scheme for the SC-CP system with the insufficient GI. In the SC-CP system, the equalization and demodulation processing is conducted in a block-by-block manner, therefore, the IBI can be reduced by using previously detected data signals. For the ISI cancellation, we firstly generate replica signals of the ISI using tentative decisions in order to make the defective channel matrix to be circulant, and then the conventional FDE is performed to compensate the ISI. As for the replica signals, we propose two tentative decision generation methods, where our newly derived FDE is utilized. We also derive minimum mean–square–error (MMSE) and zero forcing (ZF) equalizers for the shake of performance benchmark. Computer simulation results show that the proposed interference cancellation scheme can significantly improve the bit error rate (BER) performance and can outperform the MMSE equalizer while it requires lower computational complexity.

2. PROPOSED INTERFERENCE CANCELLATION SCHEME

2.1. Signal Modeling

The nth transmitted signal block $\mathbf{s}'(n)$ of size $(M+K)\times 1$ is generated from the information block $\mathbf{s}(n) = [s_0(n),\ldots,s_{M-1}(n)]^T$ by inserting the CP of K symbols length as the GI.

$$\mathbf{s}'(n) = \mathbf{T}_{cp}\mathbf{s}(n), \tag{1}$$

where $\mathbf{T}_{cp}$ denotes the $(M+K)\times M$ CP insertion matrix defined as

$$\mathbf{T}_{cp} = \begin{bmatrix}\mathbf{I}_{cp}\\ \mathbf{I}_M\end{bmatrix}, \quad \mathbf{I}_{cp} = \begin{bmatrix}\mathbf{0}_{K\times(M-K)} & \mathbf{I}_K\end{bmatrix}. \tag{2}$$

$\mathbf{0}_{K\times(M-K)}$ is a zero matrix of size $K\times(M-K)$, and $\mathbf{I}_M$ is an identity matrix of size $M\times M$.

The received signal block $\mathbf{r}'(n)$ is given by

$$\mathbf{r}'(n) = \mathbf{H}_0\mathbf{s}'(n) + \mathbf{H}_1\mathbf{s}'(n-1) + \mathbf{n}'(n), \tag{3}$$

where $\mathbf{n}'(n)$ is a channel noise vector, $\mathbf{H}_0$ and $\mathbf{H}_1$ denote $(M+K)\times(M+K)$ channel matrices defined as

$$\mathbf{H}_0 = \begin{bmatrix} h_0 & 0 & \ldots & \ldots & \ldots & 0 \\ \vdots & \ddots & \ddots & & & \vdots \\ h_L & & \ddots & \ddots & & \vdots \\ 0 & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & 0 \\ 0 & \ldots & 0 & h_L & \ldots & h_0 \end{bmatrix}, \tag{4}$$

$$\mathbf{H}_1 = \begin{bmatrix} 0 & \dots & 0 & h_L & \dots & h_1 \\ \vdots & & & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & h_L \\ \vdots & & & & & 0 \\ \vdots & & & & & \vdots \\ 0 & \dots & \dots & \dots & \dots & 0 \end{bmatrix}. \tag{5}$$

Here, $\{h_0, \dots, h_L\}$ denotes a channel impulse response.

After discarding the CP from the received signal block $\mathbf{r}'(n)$, we have the received signal block $\mathbf{r}(n)$ of size $M \times 1$,

$$\begin{aligned} \mathbf{r}(n) &= \mathbf{R}_{cp}\mathbf{r}'(n) \\ &= \mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}\mathbf{s}(n) + \mathbf{R}_{cp}\mathbf{H}_1\mathbf{T}_{cp}\mathbf{s}(n-1) + \mathbf{R}_{cp}\mathbf{n}'(n), \end{aligned} \tag{6}$$

where $\mathbf{R}_{cp}$ denotes the $M \times (M+K)$ CP discarding matrix defined by

$$\mathbf{R}_{cp} = \begin{bmatrix} \mathbf{0}_{M\times K} & \mathbf{I}_M \end{bmatrix}. \tag{7}$$

If the length of the GI is sufficient, namely, $K \geq L$, $\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}$ becomes a circulant matrix of size $M \times M$ and $\mathbf{R}_{cp}\mathbf{H}_1\mathbf{T}_{cp}$ becomes a zero matrix. Therefore, no IBI remains in the received signal and the FDE can equalize the ISI effectively. However, if the length of the GI is insufficient ($K < L$), $\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}$ is no longer a circulant matrix and $\mathbf{R}_{cp}\mathbf{H}_1\mathbf{T}_{cp}$ is no longer a zero matrix. Instead, they can be written as

$$\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp} = \begin{bmatrix} h_0 & 0 & \dots & \dots & \dots & 0 & h_K & \dots & h_1 \\ \vdots & \ddots & \ddots & & & \vdots & \vdots & & \vdots \\ \vdots & & \ddots & \ddots & & 0 & h_L & & \vdots \\ \vdots & & & \ddots & \ddots & & \ddots & \ddots & \vdots \\ h_L & & & & \ddots & \ddots & & \ddots & h_L \\ 0 & \ddots & & & & \ddots & \ddots & & 0 \\ \vdots & \ddots & \ddots & & & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & & & \ddots & 0 \\ 0 & \dots & \dots & 0 & h_L & \dots & \dots & \dots & h_0 \end{bmatrix}, \tag{8}$$

$$\mathbf{R}_{cp}\mathbf{H}_1\mathbf{T}_{cp} = \begin{bmatrix} 0 & \dots & 0 & h_L & \dots & h_{K+1} \\ \vdots & & & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & h_L \\ \vdots & & & & & 0 \\ \vdots & & & & & \vdots \\ 0 & \dots & \dots & \dots & \dots & 0 \end{bmatrix}. \tag{9}$$

If we rewrite the received signal block $\mathbf{r}(n)$ as

$$\mathbf{r}(n) = \mathbf{C}\mathbf{s}(n) - \mathbf{C}_{ISI}\mathbf{s}(n) + \mathbf{C}_{IBI}\mathbf{s}(n-1) + \mathbf{R}_{cp}\mathbf{n}'(n), \tag{10}$$

where

$$\mathbf{C} = \begin{bmatrix} h_0 & 0 & \dots & 0 & h_L & \dots & h_1 \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & h_L \\ h_L & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \dots & 0 & h_L & \dots & \dots & h_0 \end{bmatrix}, \tag{11}$$

$$\mathbf{C}_{ISI} = \begin{bmatrix} 0 & \dots & 0 & h_L & \dots & h_{K+1} & 0 & \dots & 0 \\ \vdots & & & \ddots & \ddots & \vdots & \vdots & & \vdots \\ \vdots & & & & \ddots & h_L & \vdots & & \vdots \\ \vdots & & & & & 0 & \vdots & & \vdots \\ \vdots & & & & & \vdots & \vdots & & \vdots \\ 0 & \dots & \dots & \dots & \dots & 0 & 0 & \dots & 0 \end{bmatrix}, \tag{12}$$

$$\mathbf{C}_{IBI} = \mathbf{R}_{cp}\mathbf{H}_1\mathbf{T}_{cp}, \tag{13}$$

the second and the third term in the right hand of (9) cause the ISI and the IBI, respectively, at the output of the conventional FDE.

2.2. Inter-Block Interference Cancellation

Denoting the channel noise vector $\mathbf{R}_{cp}\mathbf{n}'(n)$ as $\mathbf{n}(n)$, the received signal block after the CP removal can be written as

$$\mathbf{r}(n) = \mathbf{C}\mathbf{s}(n) - \mathbf{C}_{ISI}\mathbf{s}(n) + \mathbf{C}_{IBI}\mathbf{s}(n-1) + \mathbf{n}(n). \tag{14}$$

Since the equalization and the detection are conducted in a block-by-block manner, the IBI component $\mathbf{C}_{IBI}\mathbf{s}(n-1)$ could be cancelled by using the previously detected data vector $\tilde{\mathbf{s}}(n-1)$. In the proposed method, we cancel the IBI by subtracting $\mathbf{C}_{IBI}\tilde{\mathbf{s}}(n-1)$ from $\mathbf{r}(n)$. After the IBI cancellation, the received signal vector $\bar{\mathbf{r}}(n)$ can be written as

$$\bar{\mathbf{r}}(n) = \mathbf{r}(n) - \mathbf{C}_{IBI}\tilde{\mathbf{s}}(n-1), \tag{15}$$

$$\approx (\mathbf{C} - \mathbf{C}_{ISI})\mathbf{s}(n) + \mathbf{n}(n), \tag{16}$$

where $\approx$ becomes an equality when $\tilde{\mathbf{s}}(n-1) = \mathbf{s}(n-1)$.

2.3. Inter-Symbol Interference Cancellation

In this section, we show ISI cancellation (or equalization) methods assuming that the IBI components are completely cancelled, namely,

$$\bar{\mathbf{r}}(n) = (\mathbf{C} - \mathbf{C}_{ISI})\mathbf{s}(n) + \mathbf{n}(n), \tag{17}$$

$$= \mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}\mathbf{s}(n) + \mathbf{n}(n). \tag{18}$$

In the followings, we firstly derive ZF and MMSE equalizers, which will be benchmarks of the proposed method. And then, we derive FDE weights for the SC-CP system with insufficient GI. Finally, we describe the details of the proposed subtractive ISI cancellation method.

2.3.1. ZF Equalization
The zero-forcing equalization can be achieved by an inverse of $\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}$, therefore, the output of the ZF equalizer will be

$$\hat{\mathbf{s}}_{zf}(n) = (\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp})^{-1}\bar{\mathbf{r}}(n). \tag{19}$$

2.3.2. MMSE Equalization
The MMSE equalizer can be obtained by minimizing $E\{tr[(\hat{\mathbf{s}}(n)-\mathbf{s}(n))(\hat{\mathbf{s}}(n)-\mathbf{s}(n))^H]\}$, where $E\{\cdot\}$ and $tr[\cdot]$ denote ensemble average and trace of the matrix, respectively. By solving the minimization problem, the output of the MMSE equalizer can be given by

$$\hat{\mathbf{s}}_{mmse}(n) = (\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp})^H \cdot \left\{\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}(\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp})^H + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M\right\}^{-1}\bar{\mathbf{r}}(n), \tag{20}$$

where σ_n^2 and σ_s^2 denote the variance of the noise and the information symbols, respectively.

2.3.3. 1-tap Frequency Domain Equalization
The channel matrix $\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}$ is no more a circulant, therefore, the 1-tap FDE can not perfectly equalize the distorted received signal even when the IBIs are completely cancelled. However, the FDE is still attractive because of the simplicity of the implementation using FFT. The output of the FDE for the SC-CP system with the insufficient GI can be given by

$$\hat{\mathbf{s}}_{fde}(n) = \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\bar{\mathbf{r}}(n), \tag{21}$$

where $\mathbf{\Gamma}$ is a diagonal matrix, whose diagonal elements are $\gamma_0,\ldots,\gamma_{M-1}$, and the mth element γ_m is given by (see Appendix)

$$\gamma_m = \frac{\lambda_m^* - g_{m,m}^*}{|\lambda_m - g_{m,m}|^2 + \sum_{i=0,i\neq m}^{M-1}|g_{m,i}|^2 + \frac{\sigma_n^2}{\sigma_s^2}}, \tag{22}$$

$$g_{m,n} = \frac{1}{M}\sum_{l=0}^{L-K-1}\sum_{i=0}^{l} h_{L-i}e^{j\frac{2\pi}{M}\{n(M-L+l)-mi\}}, \tag{23}$$

$$g_{m,m} = \frac{1}{M}\sum_{l=0}^{L-K-1}\sum_{i=0}^{l} h_{L-i}e^{j\frac{2\pi}{M}m(M-L+l-i)}, \tag{24}$$

$$\sum_{m=0}^{M-1}|g_{m,n}|^2 = \frac{1}{M}\sum_{l=0}^{L-K-1}\sum_{i=0}^{l}\sum_{l'=0}^{L-K-1}|h_{L-i}|^2 e^{j\frac{2\pi}{M}n(l-l')}. \tag{25}$$

2.3.4. FDE with $\mathbf{C}_{ISI}$ cancellation
The proposed FDE require low computational complexity and can achieve better performance than the conventional FDE, however, it still suffer from performance degradation due to the defective channel matrix $\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}$. In order to further improve the performance of the FDE, we propose a $\mathbf{C}_{ISI}\mathbf{s}(n)$ cancellation scheme using tentative decision $\tilde{\tilde{\mathbf{s}}}(n) = [\tilde{\tilde{s}}_0(n),\ldots,\tilde{\tilde{s}}_{M-1}(n)]^T$. The main idea of the proposed method is that, by adding the replica of $\mathbf{C}_{ISI}\mathbf{s}(n)$ to $\bar{\mathbf{r}}(n)$, we obtain a received signal vector $\bar{\bar{\mathbf{r}}}(n)$, which is distorted only by the circulant matrix $\mathbf{C}$ in the ideal case.

$$\bar{\bar{\mathbf{r}}}(n) = \bar{\mathbf{r}}(n) + \mathbf{C}_{ISI}\tilde{\tilde{\mathbf{s}}}(n), \tag{26}$$

$$\approx \mathbf{C}\mathbf{s}(n) + \mathbf{n}(n). \tag{27}$$

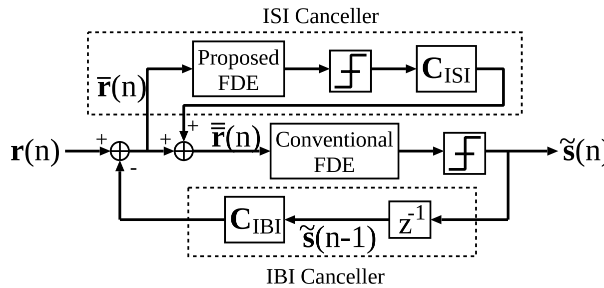

Figure 1: Proposed receiver with tentative decision 1

And then, the conventional FDE can efficiently equalize $\bar{\bar{\mathbf{r}}}(n)$ as

$$\hat{\mathbf{s}}_{cancel}(n) = \mathbf{D}^H\mathbf{\Gamma}_{cnv}\mathbf{D}\bar{\bar{\mathbf{r}}}(n), \tag{28}$$

where $\mathbf{\Gamma}_{cnv}$ is a diagonal matrix with the diagonal elements of $\gamma_0^{cnv},\ldots,\gamma_{M-1}^{cnv}$. If we employ the conventional MMSE FDE, the mth element of the equalizer can be given by

$$\gamma_m^{cnv} = \frac{\lambda_m^*}{|\lambda_m|^2 + \frac{\sigma_n^2}{\sigma_s^2}}, \tag{29}$$

where λ_m is the mth diagonal element of

$$\mathbf{\Lambda} = \mathbf{D}\mathbf{C}\mathbf{D}^H, \tag{30}$$

which is unitary similarity transformation of $\mathbf{C}$, and is calculated as

$$\begin{bmatrix}\lambda_0\\ \vdots\\ \lambda_{M-1}\end{bmatrix} = \mathbf{D}\begin{bmatrix}h_0\\ \vdots\\ h_L\\ \mathbf{0}_{1\times(M-L-1)}\end{bmatrix}. \tag{31}$$

Also, if we employ a conventional ZF FDE, γ_m^{cnv} is given by

$$\gamma_i = \frac{1}{\lambda_i}. \tag{32}$$

As for the tentative decision used for the replica signal generation, we consider two schemes as follows:

- **Tentative Decision 1**: In this scheme, we directly utilize the output of the conventional FDEs for the tentative decision, namely,

$$\tilde{\tilde{\mathbf{s}}}(n) = \tilde{\mathbf{s}}_{fde}(n) = \langle\hat{\mathbf{s}}_{fde}(n)\rangle, \tag{33}$$

  where $\langle\cdot\rangle$ stands for the detection operation. Figure 1 is the configuration of the proposed receiver using the tentative decision 1 for the replica signal generation.

- **Tentative Decision 2**: In this scheme, we utilize the structure of $\mathbf{C}_{ISI}$. Since $\mathbf{C}_{ISI}$ has nonzero elements only in $L-K$ columns,

$$\mathbf{C}_{ISI}\mathbf{s}(n) = \mathbf{C}_{ISI}\begin{bmatrix}\mathbf{0}_{(M-L)\times 1}\\ \mathbf{s}^{sub}(n)\\ \mathbf{0}_{K\times 1}\end{bmatrix} \tag{34}$$

$$= \mathbf{C}_{ISI}\mathbf{F}_s\mathbf{F}_s^T\mathbf{s}(n), \tag{35}$$

where $\mathbf{s}^{sub}(n) = [s_{M-L}(n), \ldots, s_{M-K-1}(n)]^T = \mathbf{F}_s^T \mathbf{s}(n)$ and

$$\mathbf{F}_s = \begin{bmatrix} \mathbf{0}_{(M-L)\times(L-K)} \\ \mathbf{I}_{L-K} \\ \mathbf{0}_{K\times(L-K)} \end{bmatrix}. \tag{36}$$

Therefore, only the corresponding tentative decision $\tilde{\tilde{\mathbf{s}}}^{sub}(n)$, which is defined in the same way as $\mathbf{s}^{sub}(n)$, is required in order to generate the replica of $\mathbf{C}_{ISI}\mathbf{s}(n)$. Moreover, since

$$\begin{aligned} &\mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}\mathbf{s}(n) - \mathbf{C}\left(\mathbf{I}_M - \mathbf{F}_s\mathbf{F}_s^H\right)\mathbf{s}(n) \\ &= \mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}\mathbf{s}(n) - \mathbf{C}\left(\mathbf{s}(n) - \begin{bmatrix} \mathbf{0}_{(M-L)\times 1} \\ \mathbf{s}^{sub}(n) \\ \mathbf{0}_{K\times 1} \end{bmatrix}\right) \\ &= \mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}\begin{bmatrix} \mathbf{0}_{(M-L)\times 1} \\ \mathbf{s}^{sub}(n) \\ \mathbf{0}_{K\times 1} \end{bmatrix}, \end{aligned} \tag{37}$$

we have

$$\begin{aligned} \bar{\mathbf{r}}'(n) &\stackrel{\text{def}}{=} \bar{\mathbf{r}}(n) - \mathbf{C}\left(\tilde{\mathbf{s}}_{fde}(n) - \begin{bmatrix} \mathbf{0}_{(M-L)\times 1} \\ \tilde{\mathbf{s}}_{fde}^{sub}(n) \\ \mathbf{0}_{K\times 1} \end{bmatrix}\right) \\ &\approx \mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}\begin{bmatrix} \mathbf{0}_{(M-L)\times 1} \\ \mathbf{s}^{sub}(n) \\ \mathbf{0}_{K\times 1} \end{bmatrix}, \end{aligned} \tag{38}$$

where $\tilde{\mathbf{s}}_{fde}^{sub}(n) = \mathbf{F}_s^T \tilde{\mathbf{s}}_{fde}^{sub}(n)$. Furthermore, defining

$$\mathbf{F}_r = \begin{bmatrix} \mathbf{0}_{(M-L)\times L} \\ \mathbf{I}_L \end{bmatrix}, \tag{39}$$

and $\bar{\mathbf{r}}'^{sub}(n) = \mathbf{F}_r^T \bar{\mathbf{r}}'(n)$, we finally have

$$\bar{\mathbf{r}}'^{sub}(n) \approx \mathbf{E}\mathbf{s}^{sub}(n), \tag{40}$$

where

$$\mathbf{E} = \mathbf{F}_r^T \mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}\mathbf{F}_s = \begin{bmatrix} h_0 & & \mathbf{0} \\ \vdots & \ddots & \\ \vdots & & h_0 \\ \vdots & & \vdots \\ h_{L-1} & \ldots & h_k \end{bmatrix}, \tag{41}$$

By solving the overdetermined system of (40), the tentative decision for the replica generation can be given by

$$\tilde{\tilde{\mathbf{s}}}^{sub}(n) = \langle (\mathbf{E}^H\mathbf{E})^{-1}\mathbf{E}\bar{\mathbf{r}}'^{sub}(n)\rangle. \tag{42}$$

The schematic diagram of the proposed canceller with the tentative decision 2 is shown in Figure 2.

3. COMPUTER SIMULATION

In order to confirm the validity of the proposed method, computer simulations are conducted with the following system parameters; Mod./Demod. scheme: QPSK, symbols per block: $M = 64$, GI: $K = 16$, channel order: $L = 20$, channel model: 10-path frequency selective Rayleigh fading channel. As for the systems to be evaluated, we consider following 8 systems with various equalization or interference cancellation methods:

- **Conventional FDE**: The receiver has a conventional FDE (29) and no interference canceller are employed.

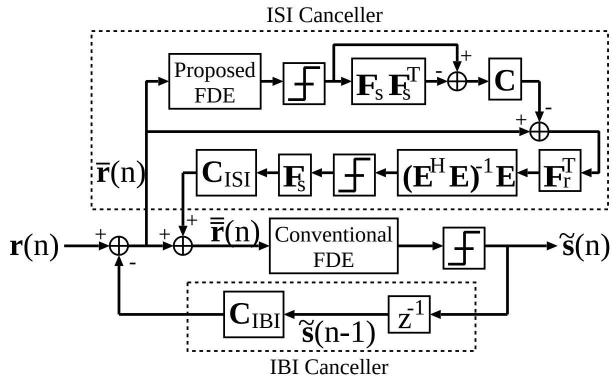

Figure 2: Proposed receiver with tentative decision 2

- **Proposed FDE**: The receiver has a proposed FDE (22), and no interference canceller is employed.
- **Proposed FDE with IBI cncl**: The proposed FDE (22) and the IBI canceller are employed.
- **Proposed FDE with IBI and ISI cncl (TD1)**: The proposed FDE (22) and the IBI and the ISI cancellers are employed. The tentative decision 1 is used for the ISI cancellation.
- **Proposed FDE with IBI and ISI cncl (TD2)**: The proposed FDE (22) and the IBI and the ISI cancellers are employed. The tentative decision 2 is used for the ISI cancellation.
- **ZF with IBI cncl**: The ZF equalizer (19) and the IBI canceller are employed.
- **MMSE with IBI cncl**: The MMSE equalizer (20) and the IBI canceller are employed.
- **MMSE with sufficient GI**: The MMSE equalizer (20) (or equivalently the conventional FDE (29)) is employed with sufficient length of the GI ($K = 20$ is used only for this receiver).

Figs.3 shows the BER performances versus the ratio of the energy per bit to the noise power density (E_b/N_0) of the above 8 systems. From the Figure, we can see that "Proposed FDE with IBI and ISI cncl (TD2)" can achieve the best performance among the systems with insufficient GI, and the performance is close to the MMSE equalizer with the sufficient GI. Amazingly enough, "Proposed FDE with IBI and ISI cncl (TD2)" can outperform even "MMSE with IBI cncl". This could be explained by the existence of a nonlinear processing in the proposed ISI canceller, namely, the detection operation. The nonlinear operation makes it possible for the proposed system to outperform the optimum MMSE linear equalizer. Moreover, even only the proposed IBI cancellation can significantly improve the BER performance.

4. CONCLUSION

We have proposed a subtractive ISI and IBI cancellation scheme for the SC-CP system with the insufficient GI. Also, the BER performances of the proposed schemes are confirmed via computer simulations. From the results, the proposed FDE with the IBI and ISI cancellation using the tentative decision 2 can outperform the MMSE equalizer with relatively low computational complexity due to the capability of the implementation using FFT.

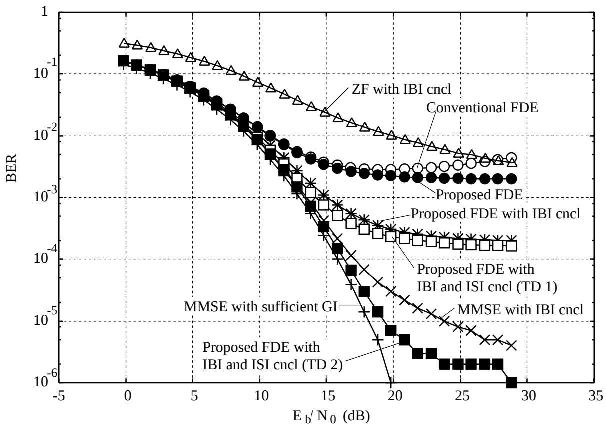

Figure 3: BER Performance

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APPENDIX

Here, we derive MMSE weights of the FDE for the SC-CP system with the insufficient GI, which is used in the proposed canceller.

Since the received signal vector can be rewritten as

$$\bar{\mathbf{r}}(n) = \mathbf{R}_{cp}\mathbf{H}_0\mathbf{T}_{cp}\mathbf{s}(n) + \mathbf{n}(n), \tag{43}$$

$$= \mathbf{D}^H\mathbf{\Lambda}\mathbf{D}\mathbf{s}(n) - \mathbf{C}_{ISI}\mathbf{s}(n) + \mathbf{n}(n), \tag{44}$$

denoting the FDE weights by a diagonal matrix $\mathbf{\Gamma}$, whose diagonal components are $\gamma_0, \ldots, \gamma_{M-1}$, the FDE output can be given by

$$\hat{\mathbf{s}}_{fde}(n) = \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\bar{\mathbf{r}}(n), \tag{45}$$

$$\begin{aligned} &= \mathbf{D}^H\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{D}\mathbf{s}(n) - \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{ISI}\mathbf{s}(n) \\ &\quad + \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\mathbf{n}(n). \end{aligned} \tag{46}$$

In order to derive the MMSE weights, we define a cost function J to be minimized as

$$\begin{aligned} J =& E\left\{tr[(\hat{\mathbf{s}}(n) - \mathbf{s}(n))(\hat{\mathbf{s}}^H(n) - \mathbf{s}^H(n))]\right\}, \\ =& tr[\sigma_s^2\{\mathbf{D}^H\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{\Lambda}^H\mathbf{\Gamma}^H\mathbf{D} - \mathbf{D}^H\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{D}\mathbf{C}_{ISI}^H\mathbf{D}^H\mathbf{\Gamma}^H\mathbf{D} \\ & - \mathbf{D}^H\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{D} - \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{ISI}\mathbf{D}^H\mathbf{\Lambda}^H\mathbf{\Gamma}^H\mathbf{D} \\ & + \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{ISI}\mathbf{C}_{ISI}^H\mathbf{D}^H\mathbf{\Gamma}^H\mathbf{D} + \mathbf{D}^H\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{ISI} \\ & - \mathbf{D}^H\mathbf{\Lambda}^H\mathbf{\Gamma}^H\mathbf{D} + \mathbf{C}_{ISI}^H\mathbf{D}^H\mathbf{\Gamma}^H\mathbf{D} + \mathbf{I}_M\} \\ & + \sigma_n^2\mathbf{D}^H\mathbf{\Gamma}\mathbf{\Gamma}^H\mathbf{D}]. \end{aligned} \tag{47}$$

Ignoring the term $tr[\sigma_s^2\mathbf{I}_M]$, which has no elements of $\mathbf{\Gamma}$, the cost function J can be redefined as

$$\begin{aligned} J =& \sigma_s^2\{tr[\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{\Lambda}^H\mathbf{\Gamma}^H] - tr[\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{D}\mathbf{C}_{ISI}^H\mathbf{D}^H\mathbf{\Gamma}^H] \\ & - tr[\mathbf{\Gamma}\mathbf{\Lambda}] - tr[\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{ISI}\mathbf{D}^H\mathbf{\Lambda}^H\mathbf{\Gamma}^H] \\ & + tr[\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{ISI}\mathbf{C}_{ISI}^H\mathbf{D}^H\mathbf{\Gamma}^H] + tr[\mathbf{\Gamma}\mathbf{D}\mathbf{C}_{ISI}\mathbf{D}^H] \\ & - tr[\mathbf{\Lambda}^H\mathbf{\Gamma}^H] + tr[\mathbf{D}\mathbf{C}_{ISI}^H\mathbf{D}^H\mathbf{\Gamma}^H]\} + \sigma_n^2 tr[\mathbf{\Gamma}\mathbf{\Gamma}^H]. \end{aligned} \tag{48}$$

Moreover, defining a matrix as

$$\mathbf{G} = \mathbf{D}\mathbf{C}_{ISI}\mathbf{D}^H, \tag{49}$$

and the (m, n) element of $\mathbf{G}$ as $g_{m,n}$ $(m, n = 0, \ldots, M-1)$, we have

$$\begin{aligned} J =& \sigma_s^2\sum_{m=0}^{M-1}(|\lambda_m|^2|\gamma_m|^2 - |\gamma_m|^2\lambda_m g_{m,m}^* - \gamma_m\lambda_m \\ & - \lambda_m^*|\gamma_m|^2 g_{m,m} + |\gamma_m|^2\sum_{i=0}^{M-1}|g_{m,i}|^2 + \gamma_m g_{m,m} \\ & - \lambda_m^*\gamma_m^* + g_{m,m}^*\gamma_m^*) + \sigma_n^2|\gamma_m|^2. \end{aligned} \tag{50}$$

The differentiation of J with respect to γ_m^* is given by

$$\begin{aligned} \frac{\partial J'}{\partial \gamma_m^*} =& \sigma_s^2\left\{|\lambda_m|^2|\gamma_m - \lambda_m g_{m,m}^*\gamma_m - \lambda_m^* g_{m,m}\gamma_m \right. \\ & \left. + \gamma_m\sum_{i=0}^{M-1}|g_{m,i}|^2 - \lambda_m^* + g_{m,m}^*\right\} + \sigma_n^2\gamma_m. \end{aligned} \tag{51}$$

By solving $\frac{\partial J'}{\partial \gamma_m^*} = 0$,

$$\begin{aligned} \gamma_m =& \frac{\lambda_m^* - g_{m,m}^*}{|\lambda_m|^2 - \lambda_m g_{m,m}^* - \lambda_m^* g_{m,m} + \sum_{i=0}^{M-1}|g_{m,i}|^2 + \frac{\sigma_n^2}{\sigma_s^2}}, \\ =& \frac{\lambda_m^* - g_{m,m}^*}{|\lambda_m - g_{m,m}|^2 + \sum_{i=0, i\neq m}^{M-1}|g_{m,i}|^2 + \frac{\sigma_n^2}{\sigma_s^2}}, \end{aligned} \tag{52}$$

where

$$g_{m,n} = \frac{1}{M}\sum_{l=0}^{L-K-1}\sum_{i=0}^{l} h_{L-i}e^{j\frac{2\pi}{M}\{n(M-L+l)-mi\}}, \tag{53}$$

$$g_{m,m} = \frac{1}{M}\sum_{l=0}^{L-K-1}\sum_{i=0}^{l} h_{L-i}e^{j\frac{2\pi}{M}m(M-L+l-i)}, \tag{54}$$

and

$$\sum_{m=0}^{M-1}|g_{m,n}|^2 = \frac{1}{M}\sum_{l=0}^{L-K-1}\sum_{i=0}^{l}\sum_{l'=0}^{L-K-1}|h_{L-i}|^2 e^{j\frac{2\pi}{M}n(l-l')}, \tag{55}$$

we obtain the MMSE weight (22).