Optimum Relay Position for Differential Amplify-and-Forward Cooperative Communications

Kazunori Hayashi[#1], *Kengo Shirai*[#2], *Thanongsak Himsoon*[*1], *W. Pam Siriwongpairat*[*2], *Ahmed K. Sadek*[*3], *K. J. Ray Liu*[*4], *and Hideaki Sakai*[#3]

[#]Graduate School of Informatics, Kyoto University
Yoshida Honmachi Sakyo-ku, Kyoto, 606-8501, JAPAN
[1]kazunori@i.kyoto-u.ac.jp [2]shirai@sys.i.kyoto-u.ac.jp [3]hsakai@i.kyoto-u.ac.jp

[*]Department of Electrical and Computer Engineering, University of Maryland
College Park, MD 20742, USA
[1]himsoon@Glue.umd.edu [2]wipawee@glue.umd.edu [3]aksadek@gmail.com [4]kjrliu@umd.edu

***Abstract*— This paper derives an optimum relay position of amplify-and-forward (AF) cooperative communications with the differential transmission. As a performance benchmark, we utilize the probability of outage, which is defined as a probability that the instantaneous signal-to-noise power ratio (SNR) becomes lower than a certain threshold. The optimum relay position is found based on the theoretical expression of the outage probability, Computer simulations are also conducted to verify the validity of the theoretical analysis.**

Keywords: cooperative communications, amplify-and-forward protocol, differential modulation

1. Introduction

Cooperative communication concept has been drawing much attention due to the potential impact on the various communications systems. So far, considerable number of studies have been made on the cooperation protocols, such as amplify-and-forward (AF) and decode-and-forward (DF)[1]-[3]. Among them, the differential transmission with the AF protocol[3] is of great interest from a view point of sensor networks[4]. This is because the AF protocol does not require any decoding processing to the relay node, and the differential transmission means that the destination node is free from the burden of the estimation of channel state information (CSI). These features are extremely suited for the sensor networks, where all the nodes should be simple and low power consumption. In the previous works, the relay node is usually assumed to exist or be predetermined, however, how to assign the relay node will be crucial problem in order to realize the cooperative communications systems. Recently, the relay-assignment method has been proposed for the DF cooperative communications based on the analysis of the optimum relay position[5], but no discussion on the optimum relay position for the AF cooperative communications has been made. In this paper, we derive the optimum relay position of the AF cooperative communications with the differential transmission. As a performance benchmark, we utilize the probability of outage, which is defined as a probability that the instantaneous signal-to-noise power ratio (SNR) becomes lower than a certain threshold. Based on the theoretical expression of the outage probability, we can find the optimum relay position. Computer simulations are also conducted to verify the validity of the theoretical analysis.

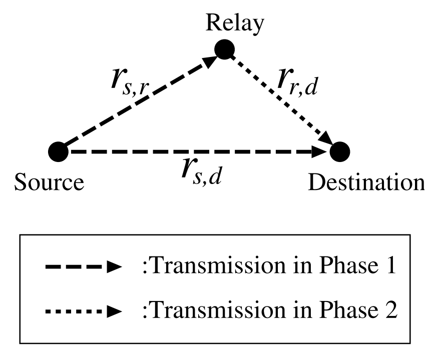

Fig. 1. System Model

2. System Model

Fig. 1 shows a system model considered in this paper. The two nodes (source and destination nodes) communicate with the AF protocol using the differential M-ary PSK modulation scheme. The information symbols transmitted by the source are written as

$$v_m = \exp(j_{\phi_m}), \tag{1}$$

where $\phi_m = 2\pi m/M$ for $m = 0, 1, \ldots, M-1$. Then, the differentially encoded transmitted signals are given by

$$x(\tau) = v_m x(\tau - 1), \quad (2)$$

where τ is the time index. In the AF protocol, the signals are transmitted in two phases, and all the users transmit signals through orthogonal channels by using a multiple access scheme, such as TDMA, FDMA or CDMA schemes[1], [2]. In Phase 1, the source node transmit the symbol $x(\tau)$ with power P. Assuming flat Rayleigh fading channels, the received signals at the destination node can be written as

$$y_{s,d}(\tau) = \sqrt{PKr_{s,d}^{-\eta}} h_{s,d}(\tau)x(\tau) + w_{s,d}(\tau), \quad (3)$$

where K is a constant that depends on the antenna design, $r_{s,d}$ is the distance between the source and the destination nodes, $h_{s,d}(\tau)$ is the channel fading gain between the nodes, which is modeled as a zero mean circularly symmetric complex Gaussian random variable with unit variance, η is the path loss exponent, and $w_{s,d}(\tau)$ is the additive white Gaussian noise (AWGN) with variance N_0. In the same way, the received signal at the relay node can be described as

$$y_{s,r}(\tau) = \sqrt{PKr_{s,r}^{-\eta}} h_{s,r}(\tau)x(\tau) + w_{s,r}(\tau). \quad (4)$$

In Phase 2, the relay node amplifies $y_{s,r}$ and transmits it to the destination with transmission power P. Here, note that, in order to verify the effect of the relay position alone, we assume the same transmission power as the source node. The received signal at the destination node is given by

$$y_{r,d}(\tau) = \sqrt{\tilde{P}Kr_{r,d}^{-\eta}} h_{r,d}(\tau)y_{s,r}(\tau) + w_{r,d}(\tau), \quad (5)$$

$$\tilde{P} = \frac{PK}{PKr_{s,r}^{-\eta} + N_0}. \quad (6)$$

The destination node combines the received signals in Phase 1 ($y_{s,d}$) and Phase 2 ($y_{r,d}$). Based on the multichannel differentially coherent detection (MDCD)[6], the combined signal is given by

$$y = a_1(y_{s,d}(\tau-1))^* y_{s,d}(\tau) + a_2(y_{r,d}(\tau-1))^* y_{r,d}(\tau), \quad (7)$$

where

$$a_1 = \frac{1}{N_0}, \quad (8)$$

$$a_2 = \frac{PKr_{s,r}^{-\eta} + N_0}{N_0(PKr_{s,r}^{-\eta} + PKr_{r,d}^{-\eta} + N_0)}. \quad (9)$$

Finally, the information signal is obtained by

$$\hat{m} = \arg \max_{m=0,1,\ldots,M-1} \Re\{v_m^* y\}. \quad (10)$$

3. Optimum Relay Position

Here, we derive optimum relay position based on the probability of outage, which is defined as

$$P_O(\gamma_{th}) = P(\gamma \leq \gamma_{th}), \quad (11)$$

where γ is the SNR of the combiner output y and γ_{th} is a threshold of outage. However, the SNR formulation of the combiner output for the weights of (9) becomes very complex and is difficult to analyze. For the simplicity, we utilize ideal maximum ratio combining weights[7]

$$\hat{a}_1 = \frac{1}{N_0}, \quad (12)$$

$$\hat{a}_2 = \frac{PKr_{s,r}^{-\eta} + N_0}{N_0(PKr_{s,r}^{-\eta} + PKr_{r,d}^{-\eta}|h_{r,d}|^2 + N_0)}, \quad (13)$$

for the derivation of the probability of outage. Note that the time index of τ in the fading gain is omitted for the notational simplicity in the hereafter. By using (13), the instantaneous SNR of the combiner output can be given by $\gamma = \gamma_1 + \gamma_2$, where

$$\gamma_1 = \frac{PK|h_{s,d}|^2 r_{s,d}^{-\eta}}{N_0}, \quad (14)$$

$$\gamma_2 = \frac{P^2K^2 r_{s,r}^{-\eta} r_{r,d}^{-\eta} |h_{s,r}|^2 |h_{r,d}|^2}{N_0(PKr_{s,r}^{-\eta} + PKr_{r,d}^{-\eta}|h_{r,d}|^2 + N_0)}. \quad (15)$$

If we have the freedom to put the relay anywhere in the two-dimensional plane, which include the source and the destination, then the optimum relay position will be on the line joining the source and the destination. This is because if the relay is at any position in the plane, then its distance to both the source and the destination are always larger than their corresponding projections on the straight line joining the source and the destination. Therefore, we can rewrite

$$r_{s,r} = m \times r_{s,d}, \quad (16)$$

and

$$r_{r,d} = (1-m) \times r_{s,d}, \quad (17)$$

where $m = [0 : 1]$. Also, since we have assumed the ideal MRC and the difference of the relay position affects not γ_1 but γ_2, the relay position which minimizes

$$P_{O\gamma_2}(\gamma_{th}) = P(\gamma_2 \leq \gamma_{th}) \quad (18)$$

also minimizes the outage probability $P_O(\gamma_{th})$. Therefore, the optimum relay position can be found via solving the following optimization problem

$$m_{opt} = \arg \min_m P_{O\gamma_2}(\gamma_{th}). \quad (19)$$

Since $|h_{s,r}|^2$ and $|h_{r,d}|^2$ are independent exponential random variables, the probability density function (PDF) of γ_2 is obtained as (see Appendix)

$$p_{\gamma_2}(\lambda) = \int_0^{\gamma_{s,d}m^{-\eta}} \frac{m^{-\eta(\gamma_{s,d}m^{-\eta}+1)}}{(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)^2 w} \cdot \exp\left(-\frac{w(\gamma_{s,d}m^{-\eta}+1)}{\gamma_{s,d}(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)} - \frac{\lambda}{w}\right)dw, \tag{20}$$

where

$$\gamma_{s,d} = \frac{PKr_{s,d}^{-\eta}}{N_0} \tag{21}$$

is the average SNR of the received signal of the direct path. Using (20), $P_{O\gamma_2}(\gamma_{th})$ is obtained as

$$\begin{aligned} P_{O\gamma_2}(\gamma_{th}) &= \int_0^{\gamma_{th}} p_{\gamma_2}(\lambda)d\lambda \quad (22)\\ &= \int_0^{\gamma_{s,d}m^{-\eta}} \frac{m^{-\eta(\gamma_{s,d}m^{-\eta}+1)}}{(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)^2} \cdot \exp\left(-\frac{w(\gamma_{s,d}m^{-\eta}+1)}{\gamma_{s,d}(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)}\right)dw \\ &\quad - \int_0^{\gamma_{s,d}m^{-\eta}} \frac{m^{-\eta(\gamma_{s,d}m^{-\eta}+1)}}{(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)^2} \cdot \exp\left(-\frac{w(\gamma_{s,d}m^{-\eta}+1)}{\gamma_{s,d}(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)} - \frac{\gamma_{th}}{w}\right)dw. \end{aligned} \tag{23}$$

While we are able to have the closed form of the first term of (23), it is difficult to have the form of the second term. So as to have the closed form of the second term, we consider the approximation of the term. By observing the argument of the exponential, we can see that γ_{th}/w is dominant for small w, while $w(\gamma_{s,d}m^{-\eta}+1)/\gamma_{s,d}(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)$ dominates the argument for large w. Therefore, the second term of (23) can be approximated (or upper bounded) as

$$\begin{aligned} &\int_0^{\gamma_{s,d}m^{-\eta}} \frac{m^{-\eta(\gamma_{s,d}m^{-\eta}+1)}}{(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)^2} \cdot \exp\left(-\frac{w(\gamma_{s,d}m^{-\eta}+1)}{\gamma_{s,d}(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)} - \frac{\gamma_{th}}{w}\right)dw \\ &\leq \int_0^{R} \frac{m^{-\eta(\gamma_{s,d}m^{-\eta}+1)}}{(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)^2} \exp\left(-\frac{\gamma_{th}}{w}\right)dw \\ &\quad + \int_R^{\gamma_{s,d}m^{-\eta}} \frac{m^{-\eta(\gamma_{s,d}m^{-\eta}+1)}}{(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)^2} \cdot \exp\left(-\frac{w(\gamma_{s,d}m^{-\eta}+1)}{\gamma_{s,d}(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)}\right)dw, \end{aligned} \tag{24}$$

where R is a certain threshold, which satisfies

$$0 \leq R \leq \gamma_{s,d}m^{-\eta}. \tag{25}$$

By using (24), $P_{O\gamma_2}(\gamma_{th})$ can be lower bounded by $P_{O\gamma_2}^{lw}(\gamma_{th})$ as

$$\begin{aligned} P_{O\gamma_2}(\gamma_{th}) &\geq P_{O\gamma_2}^{lw}(\gamma_{th}) \\ &= \int_0^{R} \frac{m^{-\eta(\gamma_{s,d}m^{-\eta}+1)}}{(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)^2} \cdot \exp\left(-\frac{w(\gamma_{s,d}m^{-\eta}+1)}{\gamma_{s,d}(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)}\right)dw \\ &\quad + \int_0^{R} \frac{m^{-\eta(\gamma_{s,d}m^{-\eta}+1)}}{(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-w)^2} \exp\left(-\frac{\gamma_{th}}{w}\right)dw \\ &= 1 - \exp\left\{-\frac{R(\gamma_{s,d}m^{-\eta}+1)}{\gamma_{s,d}(1-m)^{-\eta}(\gamma_{s,d}m^{-\eta}-R)}\right\} \\ &\quad - \frac{\gamma_{th}(\gamma_{s,d}m^{-\eta}+1)}{\gamma_{s,d}^2 m^{-\eta}(1-m)^{-\eta}} \left\{\frac{R\gamma_{s,d}m^{-\eta}\exp(-\gamma_{th}/R)}{\gamma_{th}(\gamma_{s,d}m^{-\eta}-R)} - \exp\left(-\frac{\gamma_{th}}{\gamma_{s,d}m^{-\eta}}\right)\mathrm{E}_1\left(\frac{\gamma_{th}(\gamma_{s,d}m^{-\eta}-R)}{R\gamma_{s,d}m^{-\eta}}\right)\right\}, \end{aligned} \tag{26}$$

where $\mathrm{E}_1(\cdot)$ is the exponential integral defined as[9]

$$\mathrm{E}_n(z) = \int_1^{\infty} \frac{\exp(-zt)}{t^n}dt. \tag{27}$$

Finally, we determine the threshold R in order to achieve good approximation. The approximation of (24) is an upper bound of the second term of (23), because the omitted portions, in both the first and second terms of (24), are always in the range of [0:1] for the interval of the integration. Therefore, the tightest approximation can be achieved by utilizing such R that maximizes $P_{O\gamma_2}^{lw}(\gamma_{th})$. By differentiating $P_{O\gamma_2}^{lw}(\gamma_{th})$ with respect to R and solving

$$\frac{\partial P_{O\gamma_2}^{lw}(\gamma_{th})}{\partial R} = 0, \tag{28}$$

the optimum R is obtained as

$$R_{opt} = \frac{2\gamma_{s,d}m^{-\eta}}{1+\sqrt{1+\frac{4m^{-\eta}(\gamma_{s,d}+1)}{\gamma_{th}(1-m)^{-\eta}}}}. \tag{29}$$

4. Numerical Results

Numerical calculation and computer simulations are conducted with DQPSK modulation scheme. Fig.2 shows $P_{O\gamma_2}^{lw}(\gamma_{th})$ of (26) with R of (29) versus the relay position m. The average received SNR of the direct path $\gamma_{s,d}$, the path loss exponent η, and the threshold of γ_{th} were set to be 10[dB], 3, and 3[dB], respectively. From the figure, we can see that the optimum relay position is around $m = 0.8$ in this case. This means that the optimum relay position of the AF protocol is closer to the destination than the source, while it has been shown in [5] that the optimum relay position for the DF protocol is the midpoint of the source and the destination. From a viewpoint of relay-assignment algorithm, it can be said that the nodes closer to the destination node are more suited for the relay than

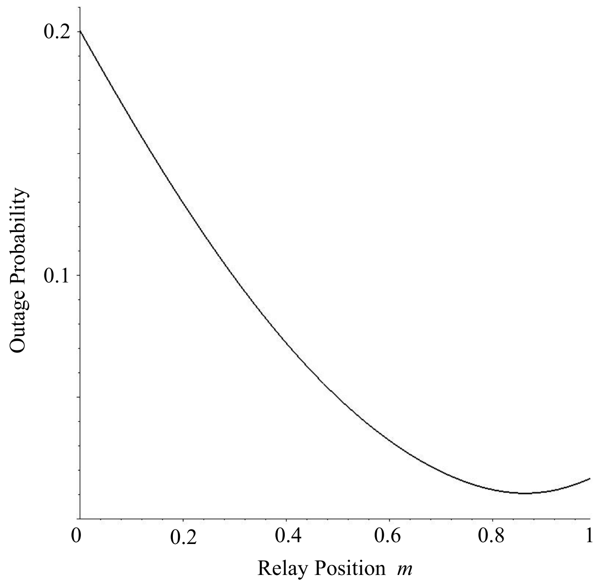

Fig. 2. Optimization of Relay Position

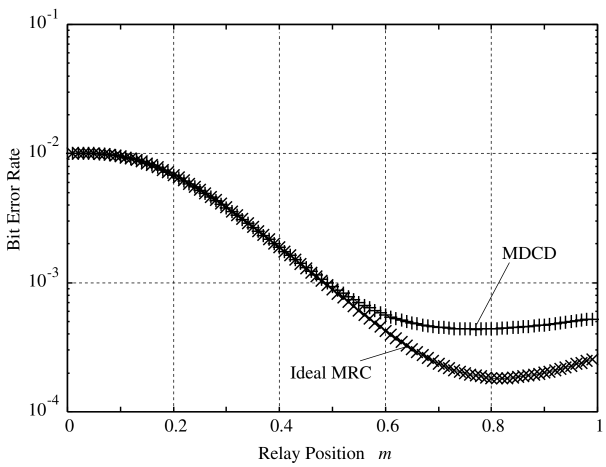

Fig. 3. Bit Error Rate Performance

the nodes closer to the source node. This asymmetry property comes from the fact that not only the desired signal but also the additive noise is also amplified at the relay node in the AF protocol. Fig. 3 shows the computer simulation results of the bit error rate (BER) performance of the MDCD with the AF protocol versus the relay position m for $\gamma_{s,d} = 20$[dB] and $\eta = 3$. The BER performance of the ideal MRC combining with the AF protocol is also plotted in the same figure. We can see that the analytical result is well accorded with the simulation results and, although we have utilized the ideal MRC in the analysis for the tractability, our analytical results can be also applied for the MDCD with the AF protocol.

5. Conclusion

We have derived the theoretical outage probability in order to optimize the relay position of the differential transmission with the AF protocol in the two-dimensional space. Unlike the DF protocol, the optimum relay position of the AF protocol is not the midpoint of the source and the destination nodes but the closer point to the destination. Also, computer simulation results are conducted to support the theoretical analysis. From all the results, it can be concluded that our analytical results can be applied for the optimization of the relay position of the MDCD with the AF protocol. The investigation of the relay assignment method for the MDCD with the AF protocol in multi-node scenario will be our future study.

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Appendix

Here, we derive the PDF of γ_2. Firstly, we define two random variables α and β as

$$\alpha = \frac{P^2 K^2 r_{s,r}^{-\eta} r_{r,d}^{-\eta} |h_{r,d}|^2}{N_0 (PK r_{s,r}^{-\eta} + PK r_{r,d}^{-\eta} |h_{r,d}|^2 + N_0)}, \quad (30)$$

$$\beta = |h_{s,r}|^2, \quad (31)$$

so that $\gamma_2 = \alpha \times \beta$. Since $|h_{s,r}|^2$ is an exponential random variable, the PDF of β is given by

$$p_\beta(\beta) = \exp(-\beta). \quad (32)$$

Defining $\zeta = |h_{r,d}|^2$, whose PDF is given by

$$p_\zeta(\zeta) = \exp(-\zeta), \quad (33)$$

α can be rewritten as

$$\alpha = \frac{P^2K^2 r_{s,r}^{-\eta} r_{r,d}^{-\eta}\zeta}{N_0(PKr_{s,r}^{-\eta} + PKr_{r,d}^{-\eta}\zeta + N_0)} \overset{\text{def}}{=} f(\zeta). \quad (34)$$

Note that the range of α is $0 \le \alpha \le \frac{PKr_{s,r}^{-\eta}}{N_0}$ because that of ζ is $0 \le \zeta \le \infty$. Here, we have

$$f^{'}(\zeta) = \frac{P^2K^2 r_{s,r}^{-\eta} r_{r,d}^{-\eta}(PKr_{s,r}^{-\eta} + N_0)}{N_0(PKr_{s,r}^{-\eta} + PKr_{r,d}^{-\eta}\zeta + N_0)^2}, \quad (35)$$

and

$$\zeta = \frac{N_0(PKr_{s,r}^{-\eta} + N_0)\alpha}{PK(PKr_{s,r}^{-\eta} r_{r,d}^{-\eta} - N_0 r_{r,d}^{-\eta}\alpha)}, \quad (36)$$

therefore, the PDF of α can be obtained as

$$\begin{aligned} p_\alpha(\alpha) &= \frac{p_\zeta(\zeta)}{|f^{'}(\zeta)|} \\ &= \frac{N_0(PKr_{s,r}^{-\eta} + PKr_{r,d}^{-\eta}\zeta + N_0)^2}{P^2K^2 r_{s,r}^{-\eta} r_{r,d}^{-\eta}(PKr_{s,r}^{-\eta} + N_0)} \exp(-\zeta) \quad (37) \\ &= \frac{N_0 r_{s,r}^{-\eta}(PKr_{s,r}^{-\eta} + N_0)}{r_{r,d}^{-\eta}(PKr_{s,r}^{-\eta} - N_0\alpha)^2} \\ &\quad \cdot \exp\left(-\frac{N_0(PKr_{s,r}^{-\eta} + N_0)\alpha}{PK(PKr_{s,r}^{-\eta} r_{r,d}^{-\eta} - N_0 r_{r,d}^{-\eta}\alpha)}\right). \quad (38) \end{aligned}$$

Since α and β are independent (because $|h_{r,d}|^2$ and $|h_{s,r}|^2$ are assumed to be independent), the joint PDF of α and β is simply given by

$$p_{\alpha\beta}(\alpha, \beta) = p_\alpha(\alpha) p_\beta(\beta). \quad (39)$$

In order to obtain the PDF of γ_2, we further consider a transformation of

$$\begin{cases} \gamma_2 = \alpha\beta, \\ \xi = \alpha. \end{cases} \quad (40)$$

Then the system has a single solution of $\alpha = \xi$, $\beta = \gamma_2/\xi$ and the Jacobian of $J(\alpha, \beta) = -\xi$. Therefore, the joint PDF of γ_2 and ξ is given by

$$\begin{aligned} p_{\gamma_2\xi}(\gamma_2, \xi) &= \frac{1}{\xi} p_{\alpha\beta}\left(\xi, \frac{\gamma_2}{\xi}\right) \\ &= \frac{N_0 r_{s,r}^{-\eta}(PKr_{s,r}^{-\eta} + N_0)}{r_{r,d}^{-\eta}(PKr_{s,r}^{-\eta} - N_0\xi)^2\xi} \\ &\quad \cdot \exp\left(-\frac{N_0(PKr_{s,r}^{-\eta} + N_0)\xi}{PK(PKr_{s,r}^{-\eta} r_{r,d}^{-\eta} - N_0 r_{r,d}^{-\eta}\xi)} - \frac{\gamma_2}{\xi}\right). \quad (41) \end{aligned}$$

Hence, the PDF of γ_2 is obtained as

$$\begin{aligned} & p_{\gamma_2}(\gamma_2) \\ &= \int_0^{\frac{PKr_{s,r}^{-\eta}}{N_0}} p_{\gamma_2\xi}(\gamma_2, \xi) dt \quad (42) \\ &= \int_0^{\frac{PKr_{s,r}^{-\eta}}{N_0}} \frac{N_0 r_{s,r}^{-\eta}(PKr_{s,r}^{-\eta} + N_0)}{r_{r,d}^{-\eta}(PKr_{s,r}^{-\eta} - N_0\xi)^2\xi} \\ &\quad \cdot \exp\left(-\frac{N_0(PKr_{s,r}^{-\eta} + N_0)\xi}{PK(PKr_{s,r}^{-\eta} r_{r,d}^{-\eta} - N_0 r_{r,d}^{-\eta}\xi)} - \frac{\gamma_2}{\xi}\right). \quad (43) \end{aligned}$$