A Power Allocation Scheme for M-QAM modulated MIMO-OFDM Systems Based On The Global BER Optimization

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***Abstract*— This paper proposes a novel transmit power allocation for QAM modulated Multi-Input Multi-Output (MIMO) Orthogonal Frequency Division Multiplexing (OFDM) transmission. This technique consists of adapting the power allocation in the spatial domain in term of the channel conditions by the optimization process based on the optimality of the global Bit Error Rate (BER). Simulation results show significant performance gain for different set of antennas and modulations.**

***Keywords*- OFDM, MIMO, Lagrangian method, global BER optimization.**

I. Introduction

The principle of OFDM transmission scheme is to divide the frequency bandwidth into small ranges and each of them is handled by low rate subcarriers where the subcarriers are orthogonal to each other. To obtain this property, the subcarrier frequencies must be spaced by a multiple of the inverse of symbol duration. Given the system description of the OFDM system, we can develop a MIMO-OFDM signal model. Principle is to apply Space Division Multiplexing (SDM) [1] on each loaded subcarrier. In addition, in this publication we will assume that communication channel remains constant during a packet transmission. The rest of the paper is organized as follows. In Section II, we will describe in detail the proposed power allocation scheme. Section III will give the simulation results over QAM modulations and finally conclusions will be drawn in Section IV.

II. System Description

A. MIMO OFDM signal

The principle of OFDM transmission scheme [**?**] is to reduce bit rate of each sub-carrier and also to provide high bit rate transmission by using a number of those low bit rate sub-carriers. Frequency bandwidth is divided into small ranges and each of them is handled by these low rate sub-carriers. Here, it is important that the sub-carriers are orthogonal to each other. To obtain this property, the sub-carrier frequencies must be spaced by a multiple of the inverse of symbol duration. Multi-carrier modulation system can provide immunity against frequency selective fading because each carrier goes through non-frequency selective fading. However, the channel must be estimated and corrected for each sub-carrier. Given the system description of the OFDM system, we can develop a MIMO-OFDM signal model. In this paper, we will need both time-domain and frequency-domain model. Suppose that a communication system consists of N_t TX and N_r RX antennas, denoted as $N_t \times N_r$ system, where the transmitter at a discrete time interval t sends a N_t-dimensional complex vector and the receiver receives an N_r-dimensional complex vector. An OFDM system transmits N modulated data symbols in the i-th OFDM symbol period through N sub-channels. The transmitted baseband OFDM signal for the i-th block symbol, is expressed as [**?**]:

$$s_{i,n}^{(p)} = \frac{1}{\sqrt{N}} . \sum_{k=0}^{N-1} x_{i,k}^{(p)} . \exp\left\{ j \frac{2\pi . nk}{N} \right\} \tag{1}$$

where $x_{i,k}^{(p)}$ is the modulated data symbol of the p-th transmit antenna for the i-th OFDM symbol. To combat ISI and Inter Carrier Interference (ICI), Guard Interval (GI) [**?**] such as Cyclic Prefix (CP) or Zero Padding (ZP) is added to the OFDM symbols. In the case of CP, the last N_g samples of every OFDM symbol are copied and added to the heading part. The transmit signal can be described as follow:

$$\tilde{s}_{i,n}^{(p)} = \begin{cases} s_{i,N-Ng+n}^{(p)}, & \text{for } 0 \le n < N_g \\ \\ s_{i,n-Ng}^{(p)}, & \text{for } N_g \le n < N + N_g \end{cases} \tag{2}$$

We assume that the system is operating in a frequency selective Rayleigh fading environment [**?**] and the communication channel remains constant during a packet transmission. Data frame duration is assumed to transmit within the coherent time of the wireless system. In this case, channel variations remain constant during on frame transmissions and may change between consecutive frame transmissions. We suppose that the fading channel can be modeled by a discrete-time baseband equivalent $(L-1)$-th order finite impulse response

(FIR) filter where L represents time samples corresponding to the maximum delay spread. In addition, an Additive White Gaussian Noise (AWGN) with N_r independent and identically distributed (iid) zero mean, complex Gaussian elements is assumed. When the maximum delay spread does not exceed GI, since ISI does not occur on MIMO OFDM symbol basis, the frequency domain MIMO OFDM signal after removal of GI is described by:

$$y_{j,m}^{(q)} = \sum_{p=0}^{N_t-1} h_m^{(q,p)}.x_{j,m}^{(p)} + n_{j,m}^{(q)} \tag{3}$$

where $y_{j,m}^{(q)}$ is the received signal at the q-th received antenna for the j-th OFDM symbol and the m-th sub-carrier and $h_{j,m}^{(q,p)}$ is the channel parameter from the p-th transmitting antenna to the q-th receiving antenna which composes the MIMO channel matrix. In addition, $n_{j,m}^{(q)}$ denotes the AWGN for the q-th received antenna. Thus it results in a frequency-flat-fading signal model per sub-carrier. For simplicity, without losing any generality, we will omit writing the index for both the sub-carrier and the symbol indicators. Hereafter, the received signal can simply be written as:

$$\mathbf{Y} = \mathbf{Hx} + \mathbf{n} \tag{4}$$

where $\mathbf{n}$ represents zero mean, complex AWGN with covariance matrix equals to:

$$\mathbf{E}[\mathbf{nn}^H] = \sigma_n^2 \mathbf{I}_{N_r} \tag{5}$$

B. MIMO OFDM detection scheme

Let now recall the linear MIMO detection with respect to the zero forcing (ZF) and to the minimum mean square error (MMSE) criteria [**?**]. In this section, we denote $\mathbf{G} = \{\mathbf{G}_m^{(l,n)}\}_{\forall l,n,m}$ the matrix representation of the detection scheme.

1) Zero Forcing Detector (ZF): In a ZF linear detector, the received signal vector is multiplied with a filter matrix which is a pseudo inverse of the channel response.

$$\mathbf{G} = (\mathbf{H}^H \mathbf{H})^{-1}.\mathbf{H}^H \tag{6}$$

2) Minimum Mean Square Detector (MMSE): The MMSE detector minimizes the mean square error between the actually transmitted symbols and the output of the linear detector which is defined by:

$$\mathbf{G} = (\alpha.\mathbf{I}_{N_r} + \mathbf{H}^H \mathbf{H})^{-1}.\mathbf{H}^H \tag{7}$$

where α is equal to $\frac{1}{SNR}$.

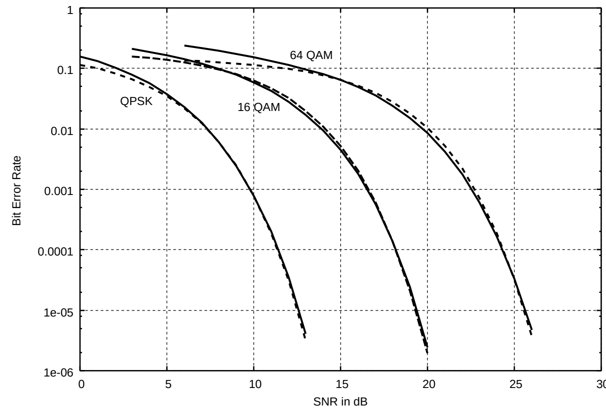

Fig. 1. Bit Error Rate for different modulations without channel coding (solid lines denote exact BER and dashed lines are fitting curves to the exact PER)

III. Power Allocation Scheme

A. Bit Error Rate approximation

For flat fading channel adhering the previous assumptions, the channel characteristics is captured by the received SNR. The received SNR is a function of the variance of the noise, the channel state information, and the transmit power. Since the channel varies from frame by frame, the approximated BER can be described as a function of the received SNR. A general formula describing this relation in a flat fading channel environment is proposed in [2]. Typically, the BER for any modulation and coding rate can be simply estimated as follows [3]:

$$f(\beta_{l,m}, p_{l,m}) \approx a \times \exp\big\{-b \times \beta_{l,m} \times p_{l,m}\big\} \tag{8}$$

with:

$$\beta_{l,m} = \frac{1}{(2^{N_m}-1).\sigma_n^2.\sum\limits_{n=0}^{N_r-1} |\mathbf{G}_m^{(l,n)}|^2} \tag{9}$$

where N_m and $\mathbf{G}_m^{(l,n)}$ respectively denote the number of bits per symbol and the complex value of the linear detection scheme between l-th transmit antenna and the n-th receive antenna on the load subcarrier m. In addition, $p_{l,m}$ represents the allocated power on the l-th transmit antenna on the m-th subcarrier. Table I summarizes the heuristic value of the parameters *a* and *b* defined in (1) for several coding rates and several modulations which are obtained by fitting (1) to simulated data.

TABLE I
Transmission modes for QAM modulations

Modulation	16-QAM	64-QAM
a	0.2	0.15
b	1.73	1.68

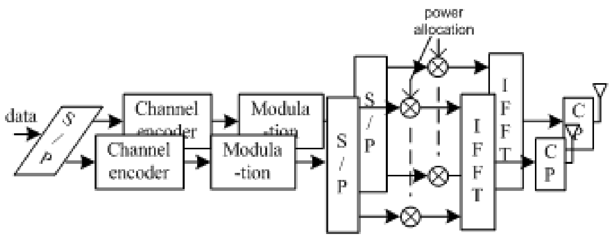

Fig. 2. Proposed spatial power allocation scheme

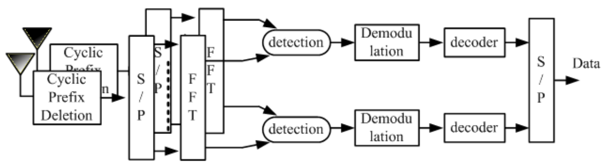

Fig. 3. MIMO-OFDM receiver scheme

B. Proposed principle

The basic principle of the proposed spatial domain power allocation for MIMO-OFDM signal is to perform spatial domain optimization of the transmit power in terms of the channel state information (CSI) and the expression of the global BER for the different modulations and coding gains. By simply performing transmit power allocation from the estimated value of the BER, we propose to optimize the transmit power allocation by performing Lagrangian method on the BER expression. Constraint is added in order to keep constant the global transmit power at the transmitting part.

Figures 1 and 2 show the proposed implementation for respectively the transmitter and the receiver parts.

The global power resource assigned by the proposed algorithm satisfies the simple relation that the total transmit power is kept constant. This constraint can be described by:

$$\sum_{l=0}^{N_t-1} p_{l,m} = N_t.\overline{P}_m \tag{10}$$

where $\overline{P}_m$ denotes the average Transmit Power on the subcarrier *m*.
The average BER becomes minimal when the BER is minimized for each given channel state. Equivalent mathematical representation can be given by:

$$\begin{cases} \min \frac{1}{N_t}. \sum_{l=0}^{N_t-1} f(\beta_{l,m}, p_{l,m}) \\ \\ \text{subject to:} \\ \sum_{l=0}^{N_t-1} p_{l,m} = N_t.\overline{P}_m \end{cases} \tag{11}$$

One possibility to solve this optimization problem is to apply the Lagrangian procedure. Defining:

$$J(p_{0,m}, ..., p_{N_t-1,m}) = \frac{1}{N_t} \sum_{l=0}^{N_t-1} f(\beta_{l,m}, p_{l,m}) + \lambda \times (\sum_{l=0}^{N_t-1} p_{l,m} - N_t \times \overline{P}_m), \tag{12}$$

the optimal solutions are obtained by solving for each transmit antenna:

$$\begin{cases} \frac{1}{N_t}.\frac{\partial}{\partial p_{l,m}}\left(\sum_{l=0}^{N_t-1} f(\beta_{l,m}, p_{l,m}) \right) + \lambda = 0 \\ \\ \sum_{l=0}^{N_t-1} p_{l,m} - N_t.\overline{P}_m = 0 \end{cases} \tag{13}$$

By introducing the explicit estimation of the BER in (8), we can rewrite the equation for each transmit antenna as:

$$\begin{cases} \frac{-a.b.\beta_{l,m}}{N_t} \times \exp(-b.\beta_{l,m}.p_{l,m}) + \lambda = 0 \\ \\ \sum_{l=0}^{N_t-1} p_{l,m} - N_t.\overline{P}_m = 0 \end{cases} \tag{14}$$

After calculation and rearrangement, we finally obtain the following general solution:

$$p_{l,m} = \left[1 + \sum_{\substack{u=0 \\ u \neq l}}^{N_t-1} \frac{\beta_{l,m}}{\beta_{u,m}} \right]^{-1} \times \left[N_t.\overline{P}_m + \frac{1}{b} \times \sum_{\substack{u=0 \\ u \neq l}}^{N_t-1} \frac{1}{\beta_{u,m}} \times \log\left(\frac{\beta_{l,m}}{\beta_{u,m}} \right) \right] \tag{15}$$

Due to the nature of the Lagrangian calculation (only mathematical solutions are obtained), we need to add a constraint when the output of the Lagrangian optimization does not reflect any physical solution, typically when we obtain: $p_{l,m} \leq 0$. In this case we propose to apply the conventional scheme, such as equal power allocation.

IV. Experimentation

We assume perfect knowledge of the channel conditions both at the transmitting and receiving parts. For all simulations, a multi-path model exponential with 1-dB decay is assumed. Carrier frequency is equal to 2.4GHz, the IFFT/FFT size is 64 points and the guard interval is set up at 16 samples. Figs.1 and 2 show the Bit Error Rate (BER) versus the average total received SNR (dB) for different antenna configurations with several modulation schemes. Both conventional and proposed schemes are plotted in these figures and results are presented for ZF detection scheme.

In Fig. 3, the benefit of performing the proposed scheme is highlighted for the case $N_t = N_r = 4$. At average BER=10^{-5}, 2dB and 2.5dB gains are respectively obtained for 16-QAM and 64-QAM modulations by simply performing the proposed scheme. For low average total SNR, for instance

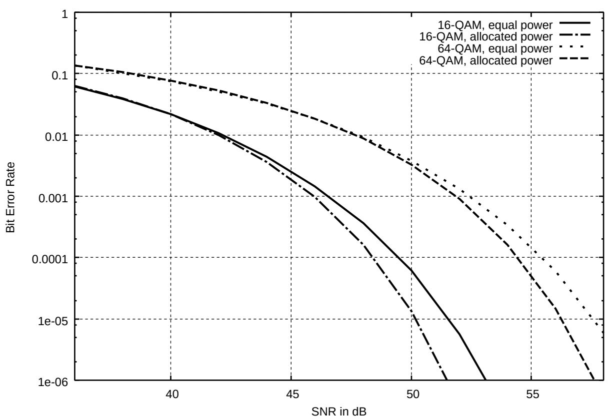

Fig. 4. Performance Simulation for Nt=Nr=4

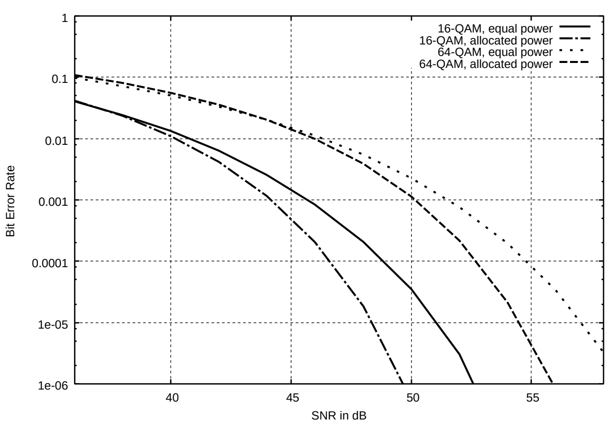

Fig. 5. Performance Simulation for Nt=Nr=8

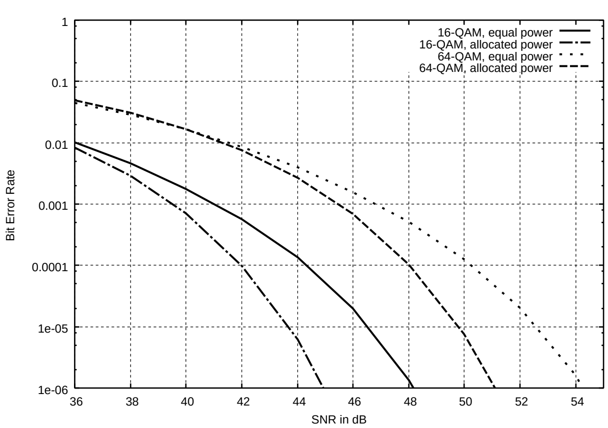

Fig. 6. Performance Simulation for Nt=Nr=12

at BER=10^{-2}, proposed scheme does not outperform the conventional scheme (equal spatial power distribution). However, from BER=5.10^{-3} impact of the proposed solution becomes significant for both 16-QAM and 64-QAM modulations. In Fig. 4, simulation results are presented the case $N_t = N_r = 8$ case. Gains obtained by performing the proposed power allocation scheme in the space domain becomes significant from BER=10^{-2} and gains about 3.5dB and 4dB are highlighted in this figure at BER=10^{-5}. In Fig. 5, impact of the proposed spatial power allocation is shown for the case $N_t = N_r = 12$ and QAM modulations. At BER=10^{-5}, several decibel gains are obtained for ZF detection.

V. Conclusion

In this paper, we propose a low complexity power allocation based on the approximated value of BER for M-QAM modulations. Simulation results have shown promising results in term of BER. In the full paper, we will describe, in detail, the proposed power allocation scheme and some additional results will be shown. Capabilities of the power allocation method for MIMO systems will be highlighted for different modulations and also for different sets of transmit and receive antennas.

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